

Mathematica 11.3 Integration Test Results

Test results for the 538 problems in "4.1.0 (a sin)^m (b trig)^{n.m"}

Problem 35: Result unnecessarily involves higher level functions.

$$\int (c \sin[a + b x])^{1/3} dx$$

Optimal (type 4, 517 leaves, 1 step):

$$-\frac{1}{b} 3 \sqrt{\frac{3}{2} \left(3 - \frac{i}{\sqrt{3}}\right)} c^{1/3}$$

$$\begin{aligned} & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{1 - \frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}}}{\sqrt{3 + \frac{i}{\sqrt{3}}}}\right], \frac{3 \frac{i}{\sqrt{3}} - \sqrt{3}}{3 \frac{i}{\sqrt{3}} + \sqrt{3}}\right] \sec[a + b x] \sqrt{1 - \frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}} \\ & \sqrt{\frac{\frac{i}{\sqrt{3}} + \sqrt{3}}{3 \frac{i}{\sqrt{3}} + \sqrt{3}} + \frac{2 (c \sin[a+b x])^{2/3}}{(3 - \frac{i}{\sqrt{3}}) c^{2/3}}} \sqrt{\frac{\frac{i}{\sqrt{3}} - \sqrt{3}}{3 \frac{i}{\sqrt{3}} - \sqrt{3}} + \frac{2 (c \sin[a+b x])^{2/3}}{(3 + \frac{i}{\sqrt{3}}) c^{2/3}}} + \frac{1}{2 \sqrt{2} b} \\ & 3 \left(1 - \frac{i}{\sqrt{3}}\right) \sqrt{3 - \frac{i}{\sqrt{3}}} c^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{1 - \frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}}}{\sqrt{3 - \frac{i}{\sqrt{3}}}}\right], \frac{3 \frac{i}{\sqrt{3}} + \sqrt{3}}{3 \frac{i}{\sqrt{3}} - \sqrt{3}}\right] \sec[a + b x] \\ & \sqrt{1 - \frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}} \sqrt{\frac{\frac{i}{\sqrt{3}} + \sqrt{3}}{3 \frac{i}{\sqrt{3}} + \sqrt{3}} + \frac{2 (c \sin[a+b x])^{2/3}}{(3 - \frac{i}{\sqrt{3}}) c^{2/3}}} \sqrt{\frac{\frac{i}{\sqrt{3}} - \sqrt{3}}{3 \frac{i}{\sqrt{3}} - \sqrt{3}} + \frac{2 (c \sin[a+b x])^{2/3}}{(3 + \frac{i}{\sqrt{3}}) c^{2/3}}} \end{aligned}$$

Result (type 5, 59 leaves):

$$-\frac{\cos[a + b x] \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \cos[a + b x]^2\right] \sin[a + b x] (c \sin[a + b x])^{1/3}}{b (\sin[a + b x]^2)^{2/3}}$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(c \sin[a + b x])^{1/3}} dx$$

Optimal (type 4, 252 leaves, 1 step):

$$\begin{aligned}
 & -\frac{1}{\sqrt{2} b c^{1/3}} 3 \sqrt{3-i \sqrt{3}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{1-\frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}}}{\sqrt{3-i \sqrt{3}}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right] \sec[a+b x] \\
 & \sqrt{1-\frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}} \sqrt{\frac{i+\sqrt{3}}{3 i+\sqrt{3}}+\frac{2 (c \sin[a+b x])^{2/3}}{(3-i \sqrt{3}) c^{2/3}}} \sqrt{\frac{i-\sqrt{3}}{3 i-\sqrt{3}}+\frac{2 (c \sin[a+b x])^{2/3}}{(3+i \sqrt{3}) c^{2/3}}}
 \end{aligned}$$

Result (type 5, 59 leaves):

$$\begin{aligned}
 & -\frac{\cos[a+b x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \cos[a+b x]^2\right] \sin[a+b x]}{b (c \sin[a+b x])^{1/3} (\sin[a+b x]^2)^{1/3}}
 \end{aligned}$$

Problem 37: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(c \sin[a+b x])^{2/3}} dx$$

Optimal (type 4, 271 leaves, 1 step):

$$\begin{aligned}
 & \left\{ 3^{3/4} \text{EllipticF}\left[\text{ArcCos}\left[\frac{c^{2/3}-\left(1-\sqrt{3}\right) (c \sin[a+b x])^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right) (c \sin[a+b x])^{2/3}}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right] \sec[a+b x] \right. \\
 & \left. (c \sin[a+b x])^{1/3} \left(c^{2/3}-\left(c \sin[a+b x]\right)^{2/3}\right) \sqrt{\frac{c^{4/3} \left(1+\frac{(c \sin[a+b x])^{2/3}}{c^{2/3}}+\frac{(c \sin[a+b x])^{4/3}}{c^{4/3}}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right) (c \sin[a+b x])^{2/3}\right)^2}} \right\} / \\
 & \left. 2 b c^{5/3} \sqrt{-\frac{\left(c \sin[a+b x]\right)^{2/3} \left(c^{2/3}-\left(c \sin[a+b x]\right)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right) (c \sin[a+b x])^{2/3}\right)^2}} \right)
 \end{aligned}$$

Result (type 5, 59 leaves):

$$\begin{aligned}
 & -\frac{\cos[a+b x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \cos[a+b x]^2\right] \sin[a+b x]}{b (c \sin[a+b x])^{2/3} (\sin[a+b x]^2)^{1/6}}
 \end{aligned}$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \cos[a+b x] \sin[a+b x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\sin[a+b x]^2}{2 b}$$

Result (type 3, 37 leaves):

$$\frac{1}{2} \left(-\frac{\cos[2a] \cos[2bx]}{2b} + \frac{\sin[2a] \sin[2bx]}{2b} \right)$$

Problem 62: Result more than twice size of optimal antiderivative.

$$\int \sin[a+bx] \tan[a+bx] dx$$

Optimal (type 3, 23 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a+bx]]}{b} - \frac{\sin[a+bx]}{b}$$

Result (type 3, 67 leaves):

$$-\frac{\log[\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)]]}{b} + \frac{\log[\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)]]}{b} - \frac{\sin[a+bx]}{b}$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int \sec[a+bx] \tan[a+bx]^2 dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\sin[a+bx]]}{2b} + \frac{\sec[a+bx] \tan[a+bx]}{2b}$$

Result (type 3, 69 leaves):

$$\frac{1}{2b} \left(\log[\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)]] - \log[\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)]] + \sec[a+bx] \tan[a+bx] \right)$$

Problem 87: Result more than twice size of optimal antiderivative.

$$\int \sec[a+bx]^4 \tan[a+bx]^4 dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{\tan[a+bx]^5}{5b} + \frac{\tan[a+bx]^7}{7b}$$

Result (type 3, 77 leaves):

$$\frac{2 \tan[a+bx]}{35b} + \frac{\sec[a+bx]^2 \tan[a+bx]}{35b} - \frac{8 \sec[a+bx]^4 \tan[a+bx]}{35b} + \frac{\sec[a+bx]^6 \tan[a+bx]}{7b}$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int \sec[a + bx]^6 \tan[a + bx]^4 dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\tan[a + bx]^5}{5b} + \frac{2\tan[a + bx]^7}{7b} + \frac{\tan[a + bx]^9}{9b}$$

Result (type 3, 98 leaves):

$$\begin{aligned} & \frac{8\tan[a + bx]}{315b} + \frac{4\sec[a + bx]^2 \tan[a + bx]}{315b} + \frac{\sec[a + bx]^4 \tan[a + bx]}{105b} - \\ & \frac{10\sec[a + bx]^6 \tan[a + bx]}{63b} + \frac{\sec[a + bx]^8 \tan[a + bx]}{9b} \end{aligned}$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int \sin[a + bx]^3 \tan[a + bx] dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{b} - \frac{\sin[a + bx]}{b} - \frac{\sin[a + bx]^3}{3b}$$

Result (type 3, 84 leaves):

$$\begin{aligned} & -\frac{\log[\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)]]}{b} + \\ & \frac{\log[\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)]]}{b} - \frac{5\sin[a + bx]}{4b} + \frac{\sin[3(a + bx)]}{12b} \end{aligned}$$

Problem 94: Result more than twice size of optimal antiderivative.

$$\int \sin[a + bx] \tan[a + bx]^3 dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$-\frac{3\operatorname{ArcTanh}[\sin[a + bx]]}{2b} + \frac{3\sin[a + bx]}{2b} + \frac{\sin[a + bx] \tan[a + bx]^2}{2b}$$

Result (type 3, 116 leaves):

$$\begin{aligned} & \frac{1}{4b} \left(6\log[\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)]] - 6\log[\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)]] + \right. \\ & \left. \frac{1}{(\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)])^2} - \frac{1}{(\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)])^2} + 4\sin[a + bx] \right) \end{aligned}$$

Problem 126: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \sec[a + bx] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\log[\tan[a + bx]]}{b}$$

Result (type 3, 31 leaves):

$$2 \left(-\frac{\log[\cos[a + bx]]}{2b} + \frac{\log[\sin[a + bx]]}{2b} \right)$$

Problem 140: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^2 \sec[a + bx] dx$$

Optimal (type 3, 23 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{b} - \frac{\csc[a + bx]}{b}$$

Result (type 3, 90 leaves):

$$-\frac{\cot\left[\frac{1}{2}(a + bx)\right]}{2b} - \frac{\log[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]]}{b} + \\ \frac{\log[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]]}{b} - \frac{\tan\left[\frac{1}{2}(a + bx)\right]}{2b}$$

Problem 142: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^2 \sec[a + bx]^3 dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}[\sin[a + bx]]}{2b} - \frac{3 \csc[a + bx]}{2b} + \frac{\csc[a + bx] \sec[a + bx]^2}{2b}$$

Result (type 3, 132 leaves):

$$\begin{aligned}
 & -\frac{1}{4 b} \left(2 \operatorname{Cot}\left[\frac{1}{2} (a + b x)\right] + 6 \operatorname{Log}\left[\cos\left[\frac{1}{2} (a + b x)\right] - \sin\left[\frac{1}{2} (a + b x)\right]\right] - \right. \\
 & \quad 6 \operatorname{Log}\left[\cos\left[\frac{1}{2} (a + b x)\right] + \sin\left[\frac{1}{2} (a + b x)\right]\right] - \frac{1}{\left(\cos\left[\frac{1}{2} (a + b x)\right] - \sin\left[\frac{1}{2} (a + b x)\right]\right)^2} + \\
 & \quad \left. \frac{1}{\left(\cos\left[\frac{1}{2} (a + b x)\right] + \sin\left[\frac{1}{2} (a + b x)\right]\right)^2} + 2 \tan\left[\frac{1}{2} (a + b x)\right] \right)
 \end{aligned}$$

Problem 144: Result more than twice size of optimal antiderivative.

$$\int \csc[a + b x]^2 \sec[a + b x]^5 dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$\frac{15 \operatorname{ArcTanh}[\sin[a + b x]]}{8 b} - \frac{15 \csc[a + b x]}{8 b} + \frac{5 \csc[a + b x] \sec[a + b x]^2}{8 b} + \frac{\csc[a + b x] \sec[a + b x]^4}{4 b}$$

Result (type 3, 219 leaves):

$$\begin{aligned}
 & -\frac{\operatorname{Cot}\left[\frac{1}{2} (a + b x)\right]}{2 b} - \frac{15 \operatorname{Log}\left[\cos\left[\frac{1}{2} (a + b x)\right] - \sin\left[\frac{1}{2} (a + b x)\right]\right]}{8 b} + \\
 & \frac{15 \operatorname{Log}\left[\cos\left[\frac{1}{2} (a + b x)\right] + \sin\left[\frac{1}{2} (a + b x)\right]\right]}{8 b} + \frac{1}{16 b \left(\cos\left[\frac{1}{2} (a + b x)\right] - \sin\left[\frac{1}{2} (a + b x)\right]\right)^4} + \\
 & \frac{7}{16 b \left(\cos\left[\frac{1}{2} (a + b x)\right] - \sin\left[\frac{1}{2} (a + b x)\right]\right)^2} - \frac{1}{16 b \left(\cos\left[\frac{1}{2} (a + b x)\right] + \sin\left[\frac{1}{2} (a + b x)\right]\right)^4} - \\
 & \frac{7}{16 b \left(\cos\left[\frac{1}{2} (a + b x)\right] + \sin\left[\frac{1}{2} (a + b x)\right]\right)^2} - \frac{\tan\left[\frac{1}{2} (a + b x)\right]}{2 b}
 \end{aligned}$$

Problem 150: Result more than twice size of optimal antiderivative.

$$\int \cot[a + b x]^2 \csc[a + b x] dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\cos[a + b x]]}{2 b} - \frac{\operatorname{Cot}[a + b x] \csc[a + b x]}{2 b}$$

Result (type 3, 75 leaves):

$$-\frac{\csc\left[\frac{1}{2} (a + b x)\right]^2}{8 b} + \frac{\operatorname{Log}\left[\cos\left[\frac{1}{2} (a + b x)\right]\right]}{2 b} - \frac{\operatorname{Log}\left[\sin\left[\frac{1}{2} (a + b x)\right]\right]}{2 b} + \frac{\sec\left[\frac{1}{2} (a + b x)\right]^2}{8 b}$$

Problem 153: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^3 \sec[a + bx]^2 dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$-\frac{3 \operatorname{ArcTanh}[\cos[a + bx]]}{2 b} + \frac{3 \sec[a + bx]}{2 b} - \frac{\csc[a + bx]^2 \sec[a + bx]}{2 b}$$

Result (type 3, 143 leaves):

$$\begin{aligned} & \left(\csc[a + bx]^4 \left(2 - 6 \cos[2(a + bx)] + 2 \cos[3(a + bx)] + \right. \right. \\ & \quad \left. \left. 3 \cos[3(a + bx)] \log[\cos[\frac{1}{2}(a + bx)]] - 3 \cos[3(a + bx)] \log[\sin[\frac{1}{2}(a + bx)]] + \right. \right. \\ & \quad \left. \left. \cos[a + bx] \left(-2 - 3 \log[\cos[\frac{1}{2}(a + bx)]] + 3 \log[\sin[\frac{1}{2}(a + bx)]] \right) \right) \right) / \\ & \quad \left(2 b \left(\csc[\frac{1}{2}(a + bx)]^2 - \sec[\frac{1}{2}(a + bx)]^2 \right) \right) \end{aligned}$$

Problem 155: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^3 \sec[a + bx]^4 dx$$

Optimal (type 3, 66 leaves, 5 steps):

$$-\frac{5 \operatorname{ArcTanh}[\cos[a + bx]]}{2 b} + \frac{5 \sec[a + bx]}{2 b} + \frac{5 \sec[a + bx]^3}{6 b} - \frac{\csc[a + bx]^2 \sec[a + bx]^3}{2 b}$$

Result (type 3, 205 leaves):

$$\begin{aligned} & \frac{1}{3 b \left(\csc[\frac{1}{2}(a + bx)]^2 - \sec[\frac{1}{2}(a + bx)]^2 \right)^3} 2 \csc[a + bx]^8 \\ & \left(22 - 40 \cos[2(a + bx)] + 13 \cos[3(a + bx)] - 30 \cos[4(a + bx)] + 13 \cos[5(a + bx)] + \right. \\ & \quad 15 \cos[3(a + bx)] \log[\cos[\frac{1}{2}(a + bx)]] + 15 \cos[5(a + bx)] \log[\cos[\frac{1}{2}(a + bx)]] - \\ & \quad 15 \cos[3(a + bx)] \log[\sin[\frac{1}{2}(a + bx)]] - 15 \cos[5(a + bx)] \log[\sin[\frac{1}{2}(a + bx)]] + \\ & \quad \left. \cos[a + bx] \left(-26 - 30 \log[\cos[\frac{1}{2}(a + bx)]] + 30 \log[\sin[\frac{1}{2}(a + bx)]] \right) \right) \end{aligned}$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \cos[a + bx]^3 \cot[a + bx]^4 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{3 \csc[a + b x]}{b} - \frac{\csc[a + b x]^3}{3 b} + \frac{3 \sin[a + b x]}{b} - \frac{\sin[a + b x]^3}{3 b}$$

Result (type 3, 121 leaves) :

$$\begin{aligned} & \frac{17 \cot\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\cot\left[\frac{1}{2}(a + b x)\right] \csc\left[\frac{1}{2}(a + b x)\right]^2}{24 b} + \frac{11 \sin[a + b x]}{4 b} + \\ & \frac{\sin[3(a + b x)]}{12 b} + \frac{17 \tan\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\sec\left[\frac{1}{2}(a + b x)\right]^2 \tan\left[\frac{1}{2}(a + b x)\right]}{24 b} \end{aligned}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int \cos[a + b x] \cot[a + b x]^4 dx$$

Optimal (type 3, 37 leaves, 3 steps) :

$$\frac{2 \csc[a + b x]}{b} - \frac{\csc[a + b x]^3}{3 b} + \frac{\sin[a + b x]}{b}$$

Result (type 3, 103 leaves) :

$$\begin{aligned} & \frac{11 \cot\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\cot\left[\frac{1}{2}(a + b x)\right] \csc\left[\frac{1}{2}(a + b x)\right]^2}{24 b} + \\ & \frac{\sin[a + b x]}{b} + \frac{11 \tan\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\sec\left[\frac{1}{2}(a + b x)\right]^2 \tan\left[\frac{1}{2}(a + b x)\right]}{24 b} \end{aligned}$$

Problem 163: Result more than twice size of optimal antiderivative.

$$\int \cot[a + b x]^3 \csc[a + b x] dx$$

Optimal (type 3, 26 leaves, 2 steps) :

$$\frac{\csc[a + b x]}{b} - \frac{\csc[a + b x]^3}{3 b}$$

Result (type 3, 93 leaves) :

$$\begin{aligned} & \frac{5 \cot\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\cot\left[\frac{1}{2}(a + b x)\right] \csc\left[\frac{1}{2}(a + b x)\right]^2}{24 b} + \\ & \frac{5 \tan\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\sec\left[\frac{1}{2}(a + b x)\right]^2 \tan\left[\frac{1}{2}(a + b x)\right]}{24 b} \end{aligned}$$

Problem 166: Result more than twice size of optimal antiderivative.

$$\int \csc[a + b x]^4 \sec[a + b x] dx$$

Optimal (type 3, 38 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{b} - \frac{\csc[a + bx]}{b} - \frac{\csc[a + bx]^3}{3b}$$

Result (type 3, 148 leaves) :

$$\begin{aligned} & -\frac{7 \cot\left[\frac{1}{2}(a + bx)\right]}{12b} - \frac{\cot\left[\frac{1}{2}(a + bx)\right] \csc\left[\frac{1}{2}(a + bx)\right]^2}{24b} - \\ & \frac{\log[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]]}{b} + \frac{\log[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]]}{b} - \\ & \frac{7 \tan\left[\frac{1}{2}(a + bx)\right]}{12b} - \frac{\sec\left[\frac{1}{2}(a + bx)\right]^2 \tan\left[\frac{1}{2}(a + bx)\right]}{24b} \end{aligned}$$

Problem 168: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^4 \sec[a + bx]^3 dx$$

Optimal (type 3, 66 leaves, 5 steps) :

$$\frac{5 \operatorname{ArcTanh}[\sin[a + bx]]}{2b} - \frac{5 \csc[a + bx]}{2b} - \frac{5 \csc[a + bx]^3}{6b} + \frac{\csc[a + bx]^3 \sec[a + bx]^2}{2b}$$

Result (type 3, 215 leaves) :

$$\begin{aligned} & -\frac{13 \cot\left[\frac{1}{2}(a + bx)\right]}{12b} - \frac{\cot\left[\frac{1}{2}(a + bx)\right] \csc\left[\frac{1}{2}(a + bx)\right]^2}{24b} - \\ & \frac{5 \log[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]]}{2b} + \frac{5 \log[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]]}{2b} + \\ & \frac{1}{1} - \frac{4b(\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right])^2}{4b(\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right])^2} - \\ & \frac{13 \tan\left[\frac{1}{2}(a + bx)\right]}{12b} - \frac{\sec\left[\frac{1}{2}(a + bx)\right]^2 \tan\left[\frac{1}{2}(a + bx)\right]}{24b} \end{aligned}$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^4 \sec[a + bx]^5 dx$$

Optimal (type 3, 89 leaves, 6 steps) :

$$\begin{aligned} & \frac{35 \operatorname{ArcTanh}[\sin[a + bx]]}{8b} - \frac{35 \csc[a + bx]}{8b} - \frac{35 \csc[a + bx]^3}{24b} + \\ & \frac{7 \csc[a + bx]^3 \sec[a + bx]^2}{8b} + \frac{\csc[a + bx]^3 \sec[a + bx]^4}{4b} \end{aligned}$$

Result (type 3, 277 leaves) :

$$\begin{aligned}
& - \frac{19 \operatorname{Cot} \left[\frac{1}{2} (a + b x) \right]}{12 b} - \frac{\operatorname{Cot} \left[\frac{1}{2} (a + b x) \right] \operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^2}{24 b} - \\
& \frac{35 \operatorname{Log} [\cos \left[\frac{1}{2} (a + b x) \right] - \sin \left[\frac{1}{2} (a + b x) \right]]}{8 b} + \frac{35 \operatorname{Log} [\cos \left[\frac{1}{2} (a + b x) \right] + \sin \left[\frac{1}{2} (a + b x) \right]]}{8 b} + \\
& \frac{16 b \left(\cos \left[\frac{1}{2} (a + b x) \right] - \sin \left[\frac{1}{2} (a + b x) \right] \right)^4}{1} + \frac{16 b \left(\cos \left[\frac{1}{2} (a + b x) \right] - \sin \left[\frac{1}{2} (a + b x) \right] \right)^2}{11} - \\
& \frac{16 b \left(\cos \left[\frac{1}{2} (a + b x) \right] + \sin \left[\frac{1}{2} (a + b x) \right] \right)^4}{1} - \frac{16 b \left(\cos \left[\frac{1}{2} (a + b x) \right] + \sin \left[\frac{1}{2} (a + b x) \right] \right)^2}{11} - \\
& \frac{19 \tan \left[\frac{1}{2} (a + b x) \right]}{12 b} - \frac{\sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right]}{24 b}
\end{aligned}$$

Problem 176: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cot} [a + b x]^4 \operatorname{Csc} [a + b x] dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$\begin{aligned}
& - \frac{3 \operatorname{ArcTanh} [\cos [a + b x]]}{8 b} + \frac{3 \operatorname{Cot} [a + b x] \operatorname{Csc} [a + b x]}{8 b} - \frac{\operatorname{Cot} [a + b x]^3 \operatorname{Csc} [a + b x]}{4 b}
\end{aligned}$$

Result (type 3, 113 leaves):

$$\begin{aligned}
& \frac{5 \operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^2}{32 b} - \frac{\operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^4}{64 b} - \frac{3 \operatorname{Log} [\cos \left[\frac{1}{2} (a + b x) \right]]}{8 b} + \\
& \frac{3 \operatorname{Log} [\sin \left[\frac{1}{2} (a + b x) \right]]}{8 b} - \frac{5 \operatorname{Sec} \left[\frac{1}{2} (a + b x) \right]^2}{32 b} + \frac{\operatorname{Sec} \left[\frac{1}{2} (a + b x) \right]^4}{64 b}
\end{aligned}$$

Problem 178: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cot} [a + b x]^2 \operatorname{Csc} [a + b x]^3 dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$\begin{aligned}
& \frac{\operatorname{ArcTanh} [\cos [a + b x]]}{8 b} + \frac{\operatorname{Cot} [a + b x] \operatorname{Csc} [a + b x]}{8 b} - \frac{\operatorname{Cot} [a + b x] \operatorname{Csc} [a + b x]^3}{4 b}
\end{aligned}$$

Result (type 3, 113 leaves):

$$\begin{aligned}
& \frac{\operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^2}{32 b} - \frac{\operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^4}{64 b} + \frac{\operatorname{Log} [\cos \left[\frac{1}{2} (a + b x) \right]]}{8 b} - \\
& \frac{\operatorname{Log} [\sin \left[\frac{1}{2} (a + b x) \right]]}{8 b} - \frac{\operatorname{Sec} \left[\frac{1}{2} (a + b x) \right]^2}{32 b} + \frac{\operatorname{Sec} \left[\frac{1}{2} (a + b x) \right]^4}{64 b}
\end{aligned}$$

Problem 181: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^5 \sec[a + bx]^2 dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$-\frac{15 \operatorname{ArcTanh}[\cos[a + bx]]}{8 b} + \frac{15 \sec[a + bx]}{8 b} -$$

$$\frac{5 \csc[a + bx]^2 \sec[a + bx]}{8 b} - \frac{\csc[a + bx]^4 \sec[a + bx]}{4 b}$$

Result (type 3, 190 leaves):

$$-\frac{7 \csc[\frac{1}{2} (a + bx)]^2}{32 b} - \frac{\csc[\frac{1}{2} (a + bx)]^4}{64 b} - \frac{15 \log[\cos[\frac{1}{2} (a + bx)]]}{8 b} +$$

$$\frac{15 \log[\sin[\frac{1}{2} (a + bx)]]}{8 b} + \frac{7 \sec[\frac{1}{2} (a + bx)]^2}{32 b} + \frac{\sec[\frac{1}{2} (a + bx)]^4}{64 b} +$$

$$\frac{\sin[\frac{1}{2} (a + bx)]}{b (\cos[\frac{1}{2} (a + bx)] - \sin[\frac{1}{2} (a + bx)])} - \frac{\sin[\frac{1}{2} (a + bx)]}{b (\cos[\frac{1}{2} (a + bx)] + \sin[\frac{1}{2} (a + bx)])}$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^5 \sec[a + bx]^4 dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$-\frac{35 \operatorname{ArcTanh}[\cos[a + bx]]}{8 b} + \frac{35 \sec[a + bx]}{8 b} +$$

$$\frac{35 \sec[a + bx]^3}{24 b} - \frac{7 \csc[a + bx]^2 \sec[a + bx]^3}{8 b} - \frac{\csc[a + bx]^4 \sec[a + bx]^3}{4 b}$$

Result (type 3, 268 leaves):

$$\begin{aligned}
& -\frac{1}{24 b \left(\csc\left[\frac{1}{2} (a+b x)\right]^2 - \sec\left[\frac{1}{2} (a+b x)\right]^2\right)^3} \csc[a+b x]^{10} \\
& \left(-204 + 658 \cos[2 (a+b x)] - 228 \cos[3 (a+b x)] + 140 \cos[4 (a+b x)] - 76 \cos[5 (a+b x)] - \right. \\
& 210 \cos[6 (a+b x)] + 76 \cos[7 (a+b x)] - 315 \cos[3 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] - \\
& 105 \cos[5 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] + 105 \cos[7 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] + \\
& 3 \cos[a+b x] \left(76 + 105 \log[\cos[\frac{1}{2} (a+b x)]] - 105 \log[\sin[\frac{1}{2} (a+b x)]] \right) + \\
& 315 \cos[3 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] + \\
& \left. 105 \cos[5 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] - 105 \cos[7 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] \right)
\end{aligned}$$

Problem 243: Result unnecessarily involves higher level functions.

$$\int (d \cos[a+b x])^{9/2} \csc[a+b x]^3 dx$$

Optimal (type 3, 113 leaves, 7 steps):

$$\begin{aligned}
& -\frac{7 d^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{d} \cos[a+b x]}{\sqrt{d}}\right]}{4 b} + \frac{7 d^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \cos[a+b x]}{\sqrt{d}}\right]}{4 b} - \\
& \frac{7 d^3 (d \cos[a+b x])^{3/2}}{6 b} - \frac{d (d \cos[a+b x])^{7/2} \csc[a+b x]^2}{2 b}
\end{aligned}$$

Result (type 5, 78 leaves):

$$\left(d^5 \left((-5 + 2 \cos[2 (a+b x)]) \cot[a+b x]^2 + \right. \right. \\
\left. \left. 21 (-\cot[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[a+b x]^2\right] \right) \right) / \left(6 b \sqrt{d \cos[a+b x]} \right)$$

Problem 245: Result unnecessarily involves higher level functions.

$$\int (d \cos[a+b x])^{5/2} \csc[a+b x]^3 dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$\begin{aligned}
& -\frac{3 d^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{d} \cos[a+b x]}{\sqrt{d}}\right]}{4 b} + \frac{3 d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \cos[a+b x]}{\sqrt{d}}\right]}{4 b} - \frac{d (d \cos[a+b x])^{3/2} \csc[a+b x]^2}{2 b}
\end{aligned}$$

Result (type 5, 65 leaves):

$$-\left(\left(d^3 \left(\cot[a+b x]^2 - 3 (-\cot[a+b x]^2)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[a+b x]^2\right] \right) \right) / \left(2 b \sqrt{d \cos[a+b x]} \right) \right)$$

Problem 246: Result unnecessarily involves higher level functions.

$$\int (d \cos[a+b x])^{3/2} \csc[a+b x]^3 dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$\frac{d^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b} + \frac{d^{3/2} \operatorname{Arctanh}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b} - \frac{d \sqrt{d \cos[a+b x]} \csc[a+b x]^2}{2 b}$$

Result (type 5, 76 leaves):

$$\begin{aligned} & \frac{1}{6 b} (d \cos[a+b x])^{3/2} (-\cot[a+b x]^2)^{3/4} \\ & \left(3 (-\cot[a+b x]^2)^{1/4} + \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc[a+b x]^2\right] \right) \sec[a+b x]^3 \end{aligned}$$

Problem 247: Result unnecessarily involves higher level functions.

$$\int \sqrt{d \cos[a+b x]} \csc[a+b x]^3 dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b} - \frac{\sqrt{d} \operatorname{Arctanh}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b} - \frac{(d \cos[a+b x])^{3/2} \csc[a+b x]^2}{2 b d}$$

Result (type 5, 62 leaves):

$$\begin{aligned} & -\left(\left(d \left(\cot[a+b x]^2 + (-\cot[a+b x]^2)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[a+b x]^2\right] \right) \right) / \left(2 b \sqrt{d \cos[a+b x]} \right) \right) \end{aligned}$$

Problem 248: Result unnecessarily involves higher level functions.

$$\int \frac{\csc[a+b x]^3}{\sqrt{d \cos[a+b x]}} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$-\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b \sqrt{d}} - \frac{3 \operatorname{Arctanh}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b \sqrt{d}} - \frac{\sqrt{d \cos[a+b x]} \csc[a+b x]^2}{2 b d}$$

Result (type 5, 69 leaves):

$$\left(d \left(-\text{Cot}[a + b x]^2\right)^{3/4} \left(\left(-\text{Cot}[a + b x]^2\right)^{1/4} - \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc[a + b x]^2\right]\right) \right) / \\ \left(2 b \left(d \cos[a + b x]\right)^{3/2} \right)$$

Problem 249: Result unnecessarily involves higher level functions.

$$\int \frac{\csc[a + b x]^3}{(d \cos[a + b x])^{3/2}} dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$\frac{5 \text{ArcTan}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b d^{3/2}} - \frac{5 \text{ArcTanh}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b d^{3/2}} + \frac{5}{2 b d \sqrt{d \cos[a+b x]}} - \frac{\csc[a+b x]^2}{2 b d \sqrt{d \cos[a+b x]}}$$

Result (type 5, 91 leaves):

$$\left(-\left(-\text{Cot}[a + b x]^2\right)^{3/4} (-4 + \text{Cot}[a + b x]^2) + \right. \\ \left. 5 \text{Cot}[a + b x]^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[a + b x]^2\right]\right) / \\ \left(2 b d \sqrt{d \cos[a + b x]} \left(-\text{Cot}[a + b x]^2\right)^{3/4}\right)$$

Problem 250: Result unnecessarily involves higher level functions.

$$\int \frac{\csc[a + b x]^3}{(d \cos[a + b x])^{5/2}} dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$-\frac{7 \text{ArcTan}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b d^{5/2}} - \frac{7 \text{ArcTanh}\left[\frac{\sqrt{d \cos[a+b x]}}{\sqrt{d}}\right]}{4 b d^{5/2}} + \\ \frac{7}{6 b d \left(d \cos[a + b x]\right)^{3/2}} - \frac{\csc[a + b x]^2}{2 b d \left(d \cos[a + b x]\right)^{3/2}}$$

Result (type 5, 92 leaves):

$$\left(\left(-\text{Cot}[a + b x]^2\right)^{1/4} (4 - 3 \text{Cot}[a + b x]^2) + \right. \\ \left. 7 \text{Cot}[a + b x]^2 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc[a + b x]^2\right]\right) / \\ \left(6 b d \left(d \cos[a + b x]\right)^{3/2} \left(-\text{Cot}[a + b x]^2\right)^{1/4}\right)$$

Problem 251: Result unnecessarily involves higher level functions.

$$\int \frac{\csc[a + b x]^3}{(d \cos[a + b x])^{7/2}} dx$$

Optimal (type 3, 137 leaves, 8 steps) :

$$\frac{9 \operatorname{ArcTan}\left[\frac{\sqrt{d} \cos [a+b x]}{\sqrt{d}}\right]}{4 b d^{7/2}} - \frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{d} \cos [a+b x]}{\sqrt{d}}\right]}{4 b d^{7/2}} +$$

$$\frac{9}{10 b d (\operatorname{d Cos}[a+b x])^{5/2}} + \frac{9}{2 b d^3 \sqrt{d} \cos [a+b x]} - \frac{\csc [a+b x]^2}{2 b d (\operatorname{d Cos}[a+b x])^{5/2}}$$

Result (type 5, 102 leaves) :

$$\left(45 \cot [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc [a+b x]^2\right] + (-\cot [a+b x]^2)^{3/4} (40 - 5 \cot [a+b x]^2 + 4 \sec [a+b x]^2)\right) /$$

$$(10 b d^3 \sqrt{d} \cos [a+b x] (-\cot [a+b x]^2)^{3/4})$$

Problem 257: Result unnecessarily involves higher level functions.

$$\int (\operatorname{d Cos}[a+b x])^{9/2} \sqrt{c \sin [a+b x]} \, dx$$

Optimal (type 4, 132 leaves, 4 steps) :

$$\frac{7 d^3 (\operatorname{d Cos}[a+b x])^{3/2} (c \sin [a+b x])^{3/2}}{30 b c} + \frac{d (\operatorname{d Cos}[a+b x])^{7/2} (c \sin [a+b x])^{3/2}}{5 b c} +$$

$$\frac{7 d^4 \sqrt{d} \cos [a+b x] \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c} \sin [a+b x]}{20 b \sqrt{\sin [2 a+2 b x]}}$$

Result (type 5, 109 leaves) :

$$\left(d^4 \sqrt{d} \cos [a+b x] \sqrt{c} \sin [a+b x]\right)$$

$$\left(-14 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos [a+b x]^2\right] \sin [2 (a+b x)] + (\sin [a+b x]^2)^{3/4} (20 \sin [2 (a+b x)] + 3 \sin [4 (a+b x)])\right) / (120 b (\sin [a+b x]^2)^{3/4})$$

Problem 258: Result unnecessarily involves higher level functions.

$$\int (\operatorname{d Cos}[a+b x])^{5/2} \sqrt{c \sin [a+b x]} \, dx$$

Optimal (type 4, 95 leaves, 3 steps) :

$$\frac{d (\operatorname{d Cos}[a+b x])^{3/2} (c \sin [a+b x])^{3/2}}{3 b c} +$$

$$\frac{d^2 \sqrt{d} \cos [a+b x] \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c} \sin [a+b x]}{2 b \sqrt{\sin [2 a+2 b x]}}$$

Result (type 5, 87 leaves) :

$$\left(d^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \right. \\ \left. - \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right) \\ \sin[2(a + b x)] \Bigg) \Bigg/ \left(6b (\sin[a + b x]^2)^{3/4} \right)$$

Problem 259: Result unnecessarily involves higher level functions.

$$\int \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} dx$$

Optimal (type 4, 53 leaves, 2 steps):

$$\frac{\sqrt{d \cos[a + b x]} \text{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{b \sqrt{\sin[2a + 2bx]}}$$

Result (type 5, 69 leaves):

$$- \left(\left(\sqrt{d \cos[a + b x]} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \right. \right. \\ \left. \left. - \sqrt{c \sin[a + b x]} \sin[2(a + b x)] \right) \Bigg/ \left(3b (\sin[a + b x]^2)^{3/4} \right) \right)$$

Problem 260: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c \sin[a + b x]}}{(d \cos[a + b x])^{3/2}} dx$$

Optimal (type 4, 93 leaves, 3 steps):

$$\frac{2(c \sin[a + b x])^{3/2}}{b c d \sqrt{d \cos[a + b x]}} - \frac{2 \sqrt{d \cos[a + b x]} \text{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{b d^2 \sqrt{\sin[2a + 2bx]}}$$

Result (type 5, 92 leaves):

$$\left(2(c \sin[a + b x])^{3/2} \right. \\ \left. - \left(2 \cos[a + b x]^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + 3(\sin[a + b x]^2)^{3/4} \right) \right) \Bigg/ \\ \left(3b c d \sqrt{d \cos[a + b x]} (\sin[a + b x]^2)^{3/4} \right)$$

Problem 261: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c \sin[a + b x]}}{(d \cos[a + b x])^{7/2}} dx$$

Optimal (type 4, 134 leaves, 4 steps):

$$\frac{\frac{2 (c \sin[a + b x])^{3/2}}{5 b c d (\cos[a + b x])^{5/2}} + \frac{4 (c \sin[a + b x])^{3/2}}{5 b c d^3 \sqrt{d \cos[a + b x]}} - \frac{4 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{5 b d^4 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 110 leaves):

$$\frac{\left(2 \sqrt{c \sin[a + b x]} \left(4 \cos[a + b x]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[a + b x] + 3 (\sin[a + b x]^2)^{3/4} (\sin[2 (a + b x)] + \tan[a + b x])\right)\right)}{\left(15 b d^2 (\cos[a + b x])^{3/2} (\sin[a + b x]^2)^{3/4}\right)}$$

Problem 262: Result unnecessarily involves higher level functions.

$$\int (\cos[a + b x])^{3/2} \sqrt{c \sin[a + b x]} dx$$

Optimal (type 3, 320 leaves, 11 steps):

$$\begin{aligned} & -\frac{\sqrt{c} d^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{4 \sqrt{2} b} + \frac{\sqrt{c} d^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{4 \sqrt{2} b} \\ & -\frac{\sqrt{c} d^{3/2} \log\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{8 \sqrt{2} b} \\ & + \frac{\sqrt{c} d^{3/2} \log\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{8 \sqrt{2} b} + \frac{d \sqrt{d \cos[a+b x]} (c \sin[a+b x])^{3/2}}{2 b c} \end{aligned}$$

Result (type 5, 82 leaves):

$$\begin{aligned} & \left((\cos[a + b x])^{3/2} \sqrt{c \sin[a + b x]} \right. \\ & \left. \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4}\right) \right. \\ & \left. \tan[a + b x]\right) \Big/ \left(2 b (\sin[a + b x]^2)^{3/4}\right) \end{aligned}$$

Problem 263: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c \sin[a + b x]}}{\sqrt{d \cos[a + b x]}} dx$$

Optimal (type 3, 280 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\sqrt{c} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{\sqrt{2} b \sqrt{d}} + \frac{\sqrt{c} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{\sqrt{2} b \sqrt{d}} + \\
& \frac{\sqrt{c} \log\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{2 \sqrt{2} b \sqrt{d}} - \\
& \frac{\sqrt{c} \log\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{2 \sqrt{2} b \sqrt{d}}
\end{aligned}$$

Result (type 5, 67 leaves):

$$-\left(\left(\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] \sqrt{c \sin[a+b x]} \sin[2(a+b x)]\right) / \right. \\
\left. \left(b \sqrt{d \cos[a+b x]} (\sin[a+b x]^2)^{3/4}\right)\right)$$

Problem 267: Result unnecessarily involves higher level functions.

$$\int (d \cos[a+b x])^{3/2} (c \sin[a+b x])^{3/2} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\begin{aligned}
& \frac{c d \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]}}{6 b} - \frac{c (d \cos[a+b x])^{5/2} \sqrt{c \sin[a+b x]}}{3 b d} + \\
& \frac{c^2 d^2 \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{12 b \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]}}
\end{aligned}$$

Result (type 5, 85 leaves):

$$-\left(\left(c d \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]} \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \cos[a+b x]^2\right] + \right.\right.\right. \\
\left.\left.\left.\cos[2(a+b x)] (\sin[a+b x]^2)^{1/4}\right)\right) / \left(6 b (\sin[a+b x]^2)^{1/4}\right)\right)$$

Problem 268: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sin[a+b x])^{3/2}}{\sqrt{d \cos[a+b x]}} dx$$

Optimal (type 4, 93 leaves, 3 steps):

$$-\frac{c \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]}}{b d} + \frac{c^2 \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{2 b \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]}}$$

Result (type 5, 67 leaves):

$$-\left(\left(\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] (\csc[a+b x])^{3/2} \sin[2(a+b x)]\right)\right) / \\ \left(b \sqrt{d \cos[a+b x]} (\sin[a+b x]^2)^{5/4}\right)$$

Problem 269: Result unnecessarily involves higher level functions.

$$\int \frac{(\csc[a+b x])^{3/2}}{(d \cos[a+b x])^{5/2}} dx$$

Optimal (type 4, 98 leaves, 3 steps):

$$\frac{2 c \sqrt{c \sin[a+b x]}}{3 b d (d \cos[a+b x])^{3/2}} - \frac{c^2 \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{3 b d^2 \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]}}$$

Result (type 5, 93 leaves):

$$\begin{aligned} & \left(2 (\csc[a+b x])^{3/2}\right. \\ & \left.\left(2 \cot[a+b x]^2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] + (\sin[a+b x]^2)^{1/4}\right)\right. \\ & \left.\left.\tan[a+b x]\right)\right) / \left(3 b d^2 \sqrt{d \cos[a+b x]} (\sin[a+b x]^2)^{1/4}\right) \end{aligned}$$

Problem 270: Result unnecessarily involves higher level functions.

$$\int \frac{(\csc[a+b x])^{3/2}}{(d \cos[a+b x])^{9/2}} dx$$

Optimal (type 4, 133 leaves, 4 steps):

$$\begin{aligned} & \frac{2 c \sqrt{c \sin[a+b x]}}{7 b d (d \cos[a+b x])^{7/2}} - \frac{2 c \sqrt{c \sin[a+b x]}}{21 b d^3 (d \cos[a+b x])^{3/2}} - \\ & \frac{2 c^2 \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{21 b d^4 \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]}} \end{aligned}$$

Result (type 5, 103 leaves):

$$\begin{aligned} & \left(2 c \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]} \left(4 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] + \right.\right. \\ & \left.\left.(-2 - \sec[a+b x]^2 + 3 \sec[a+b x]^4) (\sin[a+b x]^2)^{1/4}\right)\right) / \left(21 b d^5 (\sin[a+b x]^2)^{1/4}\right) \end{aligned}$$

Problem 271: Result unnecessarily involves higher level functions.

$$\int \sqrt{d \cos[a+b x]} (\csc[a+b x])^{3/2} dx$$

Optimal (type 3, 320 leaves, 11 steps):

$$\begin{aligned}
& \frac{\frac{c^{3/2} \sqrt{d} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{d} \sqrt{c \sin[a+b x]}}\right]}{4 \sqrt{2} b} - \frac{c^{3/2} \sqrt{d} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{d} \sqrt{c \sin[a+b x]}}\right]}{4 \sqrt{2} b}}{ \\
& \frac{c^{3/2} \sqrt{d} \log\left[\sqrt{d} + \sqrt{d} \cot[a+b x] - \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}}\right]}{8 \sqrt{2} b} + \\
& \frac{c^{3/2} \sqrt{d} \log\left[\sqrt{d} + \sqrt{d} \cot[a+b x] + \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}}\right]}{8 \sqrt{2} b} - \frac{c (d \cos[a+b x])^{3/2} \sqrt{c \sin[a+b x]}}{2 b d}}
\end{aligned}$$

Result (type 5, 80 leaves):

$$-\left(\left(c (d \cos[a+b x])^{3/2} \sqrt{c \sin[a+b x]} \left(\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] + 3 (\sin[a+b x]^2)^{1/4}\right)\right) / (6 b d (\sin[a+b x]^2)^{1/4})\right)$$

Problem 272: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sin[a+b x])^{3/2}}{(d \cos[a+b x])^{3/2}} dx$$

Optimal (type 3, 313 leaves, 11 steps):

$$\begin{aligned}
& -\frac{\frac{c^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{d} \sqrt{c \sin[a+b x]}}\right]}{\sqrt{2} b d^{3/2}} + \frac{c^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{d} \sqrt{c \sin[a+b x]}}\right]}{\sqrt{2} b d^{3/2}}} + \\
& \frac{c^{3/2} \log\left[\sqrt{d} + \sqrt{d} \cot[a+b x] - \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}}\right]}{2 \sqrt{2} b d^{3/2}} - \\
& \frac{c^{3/2} \log\left[\sqrt{d} + \sqrt{d} \cot[a+b x] + \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}}\right]}{2 \sqrt{2} b d^{3/2}} + \frac{2 c \sqrt{c \sin[a+b x]}}{b d \sqrt{d \cos[a+b x]}}
\end{aligned}$$

Result (type 5, 89 leaves):

$$\begin{aligned}
& \left(2 c \sqrt{c \sin[a+b x]} \left(\cos[a+b x]^2 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] + 3 (\sin[a+b x]^2)^{1/4}\right)\right) / \\
& (3 b d \sqrt{d \cos[a+b x]} (\sin[a+b x]^2)^{1/4})
\end{aligned}$$

Problem 276: Result unnecessarily involves higher level functions.

$$\int (d \cos[a + b x])^{9/2} (c \sin[a + b x])^{5/2} dx$$

Optimal (type 4, 166 leaves, 5 steps) :

$$\begin{aligned} & \frac{c d^3 (d \cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{20 b} + \\ & \frac{3 c d (d \cos[a + b x])^{7/2} (c \sin[a + b x])^{3/2}}{70 b} - \frac{c (d \cos[a + b x])^{11/2} (c \sin[a + b x])^{3/2}}{7 b d} + \\ & \frac{3 c^2 d^4 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{40 b \sqrt{\sin[2 a + 2 b x]}} \end{aligned}$$

Result (type 5, 122 leaves) :

$$\begin{aligned} & - \left(\left(c^2 d^4 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \right. \right. \\ & \left. \left(28 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)] + \right. \right. \\ & \left. \left. (\sin[a + b x]^2)^{3/4} (-15 \sin[2(a + b x)] + 14 \sin[4(a + b x)] + 5 \sin[6(a + b x)]) \right) \right) / \\ & \left(1120 b (\sin[a + b x]^2)^{3/4} \right) \end{aligned}$$

Problem 277: Result unnecessarily involves higher level functions.

$$\int (d \cos[a + b x])^{5/2} (c \sin[a + b x])^{5/2} dx$$

Optimal (type 4, 131 leaves, 4 steps) :

$$\begin{aligned} & \frac{c d (d \cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{10 b} - \frac{c (d \cos[a + b x])^{7/2} (c \sin[a + b x])^{3/2}}{5 b d} + \\ & \frac{3 c^2 d^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{20 b \sqrt{\sin[2 a + 2 b x]}} \end{aligned}$$

Result (type 5, 99 leaves) :

$$\begin{aligned} & - \left(\left(c^2 d^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \right. \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)] + \right. \right. \\ & \left. \left. (\sin[a + b x]^2)^{3/4} \sin[4(a + b x)] \right) \right) / \left(40 b (\sin[a + b x]^2)^{3/4} \right) \end{aligned}$$

Problem 278: Result unnecessarily involves higher level functions.

$$\int \sqrt{d \cos[a + b x]} (c \sin[a + b x])^{5/2} dx$$

Optimal (type 4, 95 leaves, 3 steps):

$$-\frac{c (d \cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{3 b d} + \frac{c^2 \sqrt{d \cos[a + b x]} \text{EllipticE}[a - \frac{\pi}{4} + b x, 2] \sqrt{c \sin[a + b x]}}{2 b \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 85 leaves):

$$-\left(\left(c^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \right. \right. \\ \left. \left. \left(\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right) \right. \right. \\ \left. \left. \sin[2(a + b x)] \right) \right) \Big/ \left(6 b (\sin[a + b x]^2)^{3/4} \right)$$

Problem 279: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sin[a + b x])^{5/2}}{(d \cos[a + b x])^{3/2}} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$\frac{2 c (c \sin[a + b x])^{3/2}}{b d \sqrt{d \cos[a + b x]}} - \frac{3 c^2 \sqrt{d \cos[a + b x]} \text{EllipticE}[a - \frac{\pi}{4} + b x, 2] \sqrt{c \sin[a + b x]}}{b d^2 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 85 leaves):

$$\left(2 c (c \sin[a + b x])^{3/2} \right. \\ \left. \left(\cos[a + b x]^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right) \right) \Big/ \\ \left(b d \sqrt{d \cos[a + b x]} (\sin[a + b x]^2)^{3/4} \right)$$

Problem 280: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sin[a + b x])^{5/2}}{(d \cos[a + b x])^{7/2}} dx$$

Optimal (type 4, 133 leaves, 4 steps):

$$\frac{2 c \left(c \sin[a+b x]\right)^{3/2}}{5 b d \left(d \cos[a+b x]\right)^{5/2}} - \frac{6 c \left(c \sin[a+b x]\right)^{3/2}}{5 b d^3 \sqrt{d \cos[a+b x]}} +$$

$$\frac{6 c^2 \sqrt{d \cos[a+b x]} \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c \sin[a+b x]}}{5 b d^4 \sqrt{\sin[2 a+2 b x]}}$$

Result (type 5, 111 leaves):

$$-\left(\left(2 c^3 \left(2 \cos[a+b x]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] + (-1+3 \cos[a+b x]^2) \left(\sin[a+b x]^2\right)^{3/4}\right) \tan[a+b x]^2\right)/\right.$$

$$\left.\left(5 b d^3 \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]} \left(\sin[a+b x]^2\right)^{3/4}\right)\right)$$

Problem 281: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sin[a+b x]\right)^{5/2}}{\left(d \cos[a+b x]\right)^{11/2}} dx$$

Optimal (type 4, 168 leaves, 5 steps):

$$\frac{2 c \left(c \sin[a+b x]\right)^{3/2}}{9 b d \left(d \cos[a+b x]\right)^{9/2}} - \frac{2 c \left(c \sin[a+b x]\right)^{3/2}}{15 b d^3 \left(d \cos[a+b x]\right)^{5/2}} - \frac{4 c \left(c \sin[a+b x]\right)^{3/2}}{15 b d^5 \sqrt{d \cos[a+b x]}} +$$

$$\frac{4 c^2 \sqrt{d \cos[a+b x]} \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c \sin[a+b x]}}{15 b d^6 \sqrt{\sin[2 a+2 b x]}}$$

Result (type 5, 119 leaves):

$$-\left(\left(2 c \sqrt{d \cos[a+b x]} \sec[a+b x]^5 \left(c \sin[a+b x]\right)^{3/2} + 4 \cos[a+b x]^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] + (-5+3 \cos[a+b x]^2+6 \cos[a+b x]^4) \left(\sin[a+b x]^2\right)^{3/4}\right)\right)/\left(45 b d^6 \left(\sin[a+b x]^2\right)^{3/4}\right)$$

Problem 282: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sin[a+b x]\right)^{5/2}}{\sqrt{d \cos[a+b x]}} dx$$

Optimal (type 3, 320 leaves, 11 steps):

$$\begin{aligned}
& - \frac{3 c^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{4 \sqrt{2} b \sqrt{d}} + \frac{3 c^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{4 \sqrt{2} b \sqrt{d}} + \\
& \frac{3 c^{5/2} \log\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{8 \sqrt{2} b \sqrt{d}} - \\
& \frac{3 c^{5/2} \log\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{8 \sqrt{2} b \sqrt{d}} - \frac{c \sqrt{d \cos[a+b x]} (c \sin[a+b x])^{3/2}}{2 b d}
\end{aligned}$$

Result (type 5, 82 leaves):

$$\begin{aligned}
& - \left(\left(\cot[a+b x] (c \sin[a+b x])^{5/2} \right. \right. \\
& \left. \left. \left(3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] + (\sin[a+b x]^2)^{3/4} \right) \right) / \\
& \left(2 b \sqrt{d \cos[a+b x]} (\sin[a+b x]^2)^{3/4} \right)
\end{aligned}$$

Problem 283: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sin[a+b x])^{5/2}}{(d \cos[a+b x])^{5/2}} dx$$

Optimal (type 3, 315 leaves, 11 steps):

$$\begin{aligned}
& \frac{c^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{\sqrt{2} b d^{5/2}} - \frac{c^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{c} \sqrt{d \cos[a+b x]}}\right]}{\sqrt{2} b d^{5/2}} - \\
& \frac{c^{5/2} \log\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{2 \sqrt{2} b d^{5/2}} + \\
& \frac{c^{5/2} \log\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+b x]}}{\sqrt{d \cos[a+b x]}} + \sqrt{c} \tan[a+b x]\right]}{2 \sqrt{2} b d^{5/2}} + \frac{2 c (c \sin[a+b x])^{3/2}}{3 b d (d \cos[a+b x])^{3/2}}
\end{aligned}$$

Result (type 5, 88 leaves):

$$\begin{aligned}
& \left(2 c (c \sin[a+b x])^{3/2} \right. \\
& \left. \left(3 \cos[a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] + (\sin[a+b x]^2)^{3/4} \right) \right) / \\
& (3 b d (d \cos[a+b x])^{3/2} (\sin[a+b x]^2)^{3/4})
\end{aligned}$$

Problem 287: Result unnecessarily involves higher level functions.

$$\int \frac{\sin[a+b x]^{7/2}}{\cos[a+b x]^{7/2}} dx$$

Optimal (type 3, 226 leaves, 12 steps):

$$\begin{aligned} & \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right]}{\sqrt{2} b} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right]}{\sqrt{2} b} - \frac{\text{Log}\left[1 + \text{Cot}[a+b x] - \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right]}{2 \sqrt{2} b} + \\ & \frac{\text{Log}\left[1 + \text{Cot}[a+b x] + \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right]}{2 \sqrt{2} b} - \frac{2 \sqrt{\sin[a+b x]}}{b \sqrt{\cos[a+b x]}} + \frac{2 \sin[a+b x]^{5/2}}{5 b \cos[a+b x]^{5/2}} \end{aligned}$$

Result (type 5, 94 leaves):

$$-\left(\left(2 \sqrt{\sin[a+b x]} \left(5 \cos[a+b x]^4 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] + 3 (2+3 \cos[2 (a+b x)]) (\sin[a+b x]^2)^{1/4}\right)\right) / \left(15 b \cos[a+b x]^{5/2} (\sin[a+b x]^2)^{1/4}\right)\right)$$

Problem 289: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\sin[x]}}{\sqrt{\cos[x]}} dx$$

Optimal (type 3, 122 leaves, 10 steps):

$$\begin{aligned} & -\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} + \\ & \frac{\text{Log}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2 \sqrt{2}} - \frac{\text{Log}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2 \sqrt{2}} \end{aligned}$$

Result (type 5, 36 leaves):

$$-\frac{2 \sqrt{\cos[x]} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[x]^2\right] \sin[x]^{3/2}}{(\sin[x]^2)^{3/4}}$$

Problem 290: Result unnecessarily involves higher level functions.

$$\int \frac{\sin[x]^{5/2}}{\sqrt{\cos[x]}} dx$$

Optimal (type 3, 143 leaves, 11 steps):

$$\begin{aligned}
 & -\frac{3 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{\sin [x]}}{\sqrt{\cos [x]}}\right]}{4 \sqrt{2}}+\frac{3 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{\sin [x]}}{\sqrt{\cos [x]}}\right]}{4 \sqrt{2}}+ \\
 & \frac{3 \log \left[1-\frac{\sqrt{2} \sqrt{\sin [x]}}{\sqrt{\cos [x]}}+\tan [x]\right]}{8 \sqrt{2}}-\frac{3 \log \left[1+\frac{\sqrt{2} \sqrt{\sin [x]}}{\sqrt{\cos [x]}}+\tan [x]\right]}{8 \sqrt{2}}-\frac{1}{2} \sqrt{\cos [x]} \sin [x]^{3 / 2}
 \end{aligned}$$

Result (type 5, 49 leaves):

$$-\frac{1}{2 (\sin [x]^2)^{3 / 4}} \sqrt{\cos [x]} \sin [x]^{3 / 2} \left(3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos [x]^2\right]+\left(\sin [x]^2\right)^{3 / 4}\right)$$

Problem 291: Result unnecessarily involves higher level functions.

$$\int \frac{(d \cos [a+b x])^{7 / 2}}{\sqrt{c \sin [a+b x]}} d x$$

Optimal (type 4, 132 leaves, 4 steps):

$$\begin{aligned}
 & \frac{5 d^3 \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}{6 b c}+ \\
 & \frac{d (d \cos [a+b x])^{5 / 2} \sqrt{c \sin [a+b x]}}{3 b c}+\frac{5 d^4 \operatorname{EllipticF}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{\sin [2 a+2 b x]}}{12 b \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}
 \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
 & -\left(\left(d^3 \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}\right.\right. \\
 & \left.\left.-30 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos [a+b x]^2\right]+25 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4},\right.\right. \\
 & \left.\left.\cos [a+b x]^2\right]+6 \cos [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{5}{4}, \frac{9}{4}, \cos [a+b x]^2\right]-\right. \\
 & \left.\left.5 \cos [2 (a+b x)] (\sin [a+b x]^2)^{1 / 4}\right)\right) /\left(30 b c (\sin [a+b x]^2)^{1 / 4}\right)
 \end{aligned}$$

Problem 292: Result unnecessarily involves higher level functions.

$$\int \frac{(d \cos [a+b x])^{3 / 2}}{\sqrt{c \sin [a+b x]}} d x$$

Optimal (type 4, 92 leaves, 3 steps):

$$\begin{aligned}
 & \frac{d \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}{b c}+\frac{d^2 \operatorname{EllipticF}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{\sin [2 a+2 b x]}}{2 b \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}
 \end{aligned}$$

Result (type 5, 69 leaves):

$$-\left(\left(\left(d \cos[a + b x] \right)^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{5}{4}, \frac{9}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)] \right) / \right. \\ \left. \left(5 b \sqrt{c \sin[a + b x]} (\sin[a + b x]^2)^{1/4} \right) \right)$$

Problem 293: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]}} dx$$

Optimal (type 4, 53 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{b \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]}}$$

Result (type 5, 67 leaves):

$$-\frac{\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)]}{b \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} (\sin[a + b x]^2)^{1/4}}$$

Problem 294: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \cos[a + b x])^{5/2} \sqrt{c \sin[a + b x]}} dx$$

Optimal (type 4, 97 leaves, 3 steps):

$$\frac{2 \sqrt{c \sin[a + b x]}}{3 b c d (d \cos[a + b x])^{3/2}} + \frac{2 \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{3 b d^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]}}$$

Result (type 5, 104 leaves):

$$\left(2 \left(-4 \cos[a + b x]^2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a + b x]^2\right] + \right. \right. \\ \left. \left. (2 + \cos[2(a + b x)]) (\sin[a + b x]^2)^{1/4} \right) \tan[a + b x] \right) / \\ \left(3 b d^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} (\sin[a + b x]^2)^{1/4} \right)$$

Problem 295: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \cos[a + b x])^{9/2} \sqrt{c \sin[a + b x]}} dx$$

Optimal (type 4, 134 leaves, 4 steps):

$$\frac{2 \sqrt{c \sin[a+b x]}}{7 b c d (\cos[a+b x])^{7/2}} + \frac{4 \sqrt{c \sin[a+b x]}}{7 b c d^3 (\cos[a+b x])^{3/2}} + \\ \frac{4 \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin[2 a + 2 b x]}}{7 b d^4 \sqrt{\cos[a+b x]} \sqrt{c \sin[a+b x]}}$$

Result (type 5, 103 leaves):

$$\left(2 \sqrt{d \cos[a+b x]} \sqrt{c \sin[a+b x]} \left(-8 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] + (4 + 2 \sec[a+b x]^2 + \sec[a+b x]^4) (\sin[a+b x]^2)^{1/4}\right)\right) / (7 b c d^5 (\sin[a+b x]^2)^{1/4})$$

Problem 296: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}} dx$$

Optimal (type 3, 280 leaves, 10 steps):

$$\frac{\sqrt{d} \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{d} \sqrt{c \sin[a+b x]}}\right] - \sqrt{d} \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{d} \sqrt{c \sin[a+b x]}}\right]}{\sqrt{2} b \sqrt{c}} - \\ \frac{\sqrt{d} \text{Log}\left[\sqrt{d} + \sqrt{d} \cot[a+b x] - \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}}\right]}{2 \sqrt{2} b \sqrt{c}} + \\ \frac{\sqrt{d} \text{Log}\left[\sqrt{d} + \sqrt{d} \cot[a+b x] + \frac{\sqrt{2} \sqrt{c} \sqrt{d \cos[a+b x]}}{\sqrt{c \sin[a+b x]}}\right]}{2 \sqrt{2} b \sqrt{c}}$$

Result (type 5, 69 leaves):

$$-\left(\left(\sqrt{d \cos[a+b x]} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] \sin[2(a+b x)]\right) / (3 b \sqrt{c \sin[a+b x]} (\sin[a+b x]^2)^{1/4})\right)$$

Problem 300: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}} dx$$

Optimal (type 3, 174 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right] - \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right]}{\sqrt{2} b} - \frac{\text{Log}\left[1 + \text{Cot}[a+b x] - \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right] - \text{Log}\left[1 + \text{Cot}[a+b x] + \frac{\sqrt{2} \sqrt{\cos[a+b x]}}{\sqrt{\sin[a+b x]}}\right]}{2 \sqrt{2} b}$$

Result (type 5, 57 leaves):

$$-\left(\left(2 \cos[a+b x]^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+b x]^2\right] \sqrt{\sin[a+b x]}\right) / \left(3 b (\sin[a+b x]^2)^{1/4}\right)\right)$$

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[a+b x]^{3/2}}{\sin[a+b x]^{3/2}} dx$$

Optimal (type 3, 199 leaves, 11 steps):

$$\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[a+b x]}}{\sqrt{\cos[a+b x]}}\right] - \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[a+b x]}}{\sqrt{\cos[a+b x]}}\right]}{\sqrt{2} b} - \frac{\text{Log}\left[1 - \frac{\sqrt{2} \sqrt{\sin[a+b x]}}{\sqrt{\cos[a+b x]}} + \tan[a+b x]\right] - \text{Log}\left[1 + \frac{\sqrt{2} \sqrt{\sin[a+b x]}}{\sqrt{\cos[a+b x]}} + \tan[a+b x]\right]}{2 \sqrt{2} b} - \frac{2 \sqrt{\cos[a+b x]}}{b \sqrt{\sin[a+b x]}}$$

Result (type 5, 78 leaves):

$$-\left(\left(2 \sqrt{\cos[a+b x]} \left(-\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+b x]^2\right] \sin[a+b x]^2 + (\sin[a+b x]^2)^{3/4}\right)\right) / \left(b \sqrt{\sin[a+b x]} (\sin[a+b x]^2)^{3/4}\right)\right)$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[a+b x]^{5/2}}{\sin[a+b x]^{5/2}} dx$$

Optimal (type 3, 201 leaves, 11 steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{\cos [a+b x]}}{\sqrt{\sin [a+b x]}}\right]}{\sqrt{2} b}+\frac{\operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{\cos [a+b x]}}{\sqrt{\sin [a+b x]}}\right]}{\sqrt{2} b}+ \\
& \frac{\operatorname{Log}\left[1+\cot [a+b x]-\frac{\sqrt{2} \sqrt{\cos [a+b x]}}{\sqrt{\sin [a+b x]}}\right]}{2 \sqrt{2} b}-\frac{\operatorname{Log}\left[1+\cot [a+b x]+\frac{\sqrt{2} \sqrt{\cos [a+b x]}}{\sqrt{\sin [a+b x]}}\right]}{2 \sqrt{2} b}-\frac{2 \cos [a+b x]^{3/2}}{3 b \sin [a+b x]^{3/2}}
\end{aligned}$$

Result (type 5, 80 leaves):

$$\begin{aligned}
& -\left(\left(2 \cos [a+b x]^{3/2}\right.\right. \\
& \left.\left.-\text { Hypergeometric } 2 F 1\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos [a+b x]^2\right] \sin [a+b x]^2+\left(\sin [a+b x]^2\right)^{1/4}\right)\right) / \\
& \left(3 b \sin [a+b x]^{3/2}\left(\sin [a+b x]^2\right)^{1/4}\right)
\end{aligned}$$

Problem 303: Result unnecessarily involves higher level functions.

$$\int \frac{\cos [a+b x]^{7/2}}{\sin [a+b x]^{7/2}} dx$$

Optimal (type 3, 226 leaves, 12 steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{\sin [a+b x]}}{\sqrt{\cos [a+b x]}}\right]}{\sqrt{2} b}+\frac{\operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{\sin [a+b x]}}{\sqrt{\cos [a+b x]}}\right]}{\sqrt{2} b}+\frac{\operatorname{Log}\left[1-\frac{\sqrt{2} \sqrt{\sin [a+b x]}}{\sqrt{\cos [a+b x]}}+\tan [a+b x]\right]}{2 \sqrt{2} b}- \\
& \frac{\operatorname{Log}\left[1+\frac{\sqrt{2} \sqrt{\sin [a+b x]}}{\sqrt{\cos [a+b x]}}+\tan [a+b x]\right]}{2 \sqrt{2} b}-\frac{2 \cos [a+b x]^{5/2}}{5 b \sin [a+b x]^{5/2}}+\frac{2 \sqrt{\cos [a+b x]}}{b \sqrt{\sin [a+b x]}}
\end{aligned}$$

Result (type 5, 93 leaves):

$$\begin{aligned}
& -\left(\left(2 \sqrt{\cos [a+b x]}\left(5 \text { Hypergeometric } 2 F 1\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos [a+b x]^2\right] \sin [a+b x]^4+\right.\right.\right. \\
& \left.\left.\left.\left(\sin [a+b x]^2\right)^{3/4}\left(1-6 \sin [a+b x]^2\right)\right)\right) /\left(5 b \sin [a+b x]^{5/2}\left(\sin [a+b x]^2\right)^{3/4}\right)
\end{aligned}$$

Problem 324: Result unnecessarily involves higher level functions.

$$\int \frac{\sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}} dx$$

Optimal (type 3, 128 leaves, 8 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\frac{1-2 \sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}}{\sqrt{3}}\right]}{2 b}-\frac{\operatorname{Log}\left[1+\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}\right]}{2 b}+\frac{\operatorname{Log}\left[1-\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}+\frac{\sin [a+b x]^{4/3}}{\cos [a+b x]^{4/3}}\right]}{4 b}$$

Result (type 5, 57 leaves):

$$-\left(\left(3 \cos [a+b x]^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \cos [a+b x]^2\right] \sin [a+b x]^{4/3}\right) / \right. \\ \left.\left(2 b (\sin [a+b x]^2)^{2/3}\right)\right)$$

Problem 325: Result unnecessarily involves higher level functions.

$$\int \frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}} dx$$

Optimal (type 3, 224 leaves, 11 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-\frac{2 \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}\right]}{2 b}+\frac{\text{ArcTan}\left[\sqrt{3}+\frac{2 \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}\right]}{2 b}+\frac{\text{ArcTan}\left[\frac{\sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}\right]}{b}+ \\ \frac{\sqrt{3} \log \left[1-\frac{\sqrt{3} \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}+\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}\right]}{4 b}-\frac{\sqrt{3} \log \left[1+\frac{\sqrt{3} \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}+\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}\right]}{4 b}$$

Result (type 5, 55 leaves):

$$-\left(\left(3 \cos [a+b x]^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \cos [a+b x]^2\right] \sin [a+b x]^{5/3}\right) / \right. \\ \left.\left(b (\sin [a+b x]^2)^{5/6}\right)\right)$$

Problem 326: Result unnecessarily involves higher level functions.

$$\int \frac{\sin [a+b x]^{4/3}}{\cos [a+b x]^{4/3}} dx$$

Optimal (type 3, 249 leaves, 12 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-\frac{2 \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{2 b}+\frac{\text{ArcTan}\left[\sqrt{3}+\frac{2 \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{2 b}+ \\ \frac{\text{ArcTan}\left[\frac{\cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{b}+\frac{\sqrt{3} \log \left[1+\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}-\frac{\sqrt{3} \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{4 b}- \\ \frac{\sqrt{3} \log \left[1+\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}+\frac{\sqrt{3} \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{4 b}+\frac{3 \sin [a+b x]^{1/3}}{b \cos [a+b x]^{1/3}}$$

Result (type 5, 83 leaves):

$$\frac{3 \sin [a+b x]^{1/3}}{b \cos [a+b x]^{1/3}}+\left(3 \cos [a+b x]^{5/3} \text{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \cos [a+b x]^2\right] \sin [a+b x]^{1/3}\right) / \\ \left(5 b (\sin [a+b x]^2)^{1/6}\right)$$

Problem 327: Result unnecessarily involves higher level functions.

$$\int \frac{\sin[a + bx]^{5/3}}{\cos[a + bx]^{5/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$\begin{aligned} & -\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\frac{1-2 \cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} + \\ & \frac{\log \left[1+\frac{\cos [a+b x]^{4/3}}{\sin [a+b x]^{4/3}}-\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}\right]}{4 b}-\frac{\log \left[1+\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}\right]}{2 b}+\frac{3 \sin [a+b x]^{2/3}}{2 b \cos [a+b x]^{2/3}} \end{aligned}$$

Result (type 5, 81 leaves):

$$\begin{aligned} & \left(3 \sin [a+b x]^{2/3}\right. \\ & \left.\left(\cos [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \cos [a+b x]^2\right]+2\left(\sin [a+b x]^2\right)^{1/3}\right)\right) / \\ & \left(4 b \cos [a+b x]^{2/3}\left(\sin [a+b x]^2\right)^{1/3}\right) \end{aligned}$$

Problem 328: Result unnecessarily involves higher level functions.

$$\int \frac{\sin[a + bx]^{7/3}}{\cos[a + bx]^{7/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$\begin{aligned} & \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\frac{1-2 \sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}}{\sqrt{3}}\right]}{2 b}+\frac{\log \left[1+\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}\right]}{2 b}- \\ & \frac{\log \left[1-\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}+\frac{\sin [a+b x]^{4/3}}{\cos [a+b x]^{4/3}}\right]}{4 b}+\frac{3 \sin [a+b x]^{4/3}}{4 b \cos [a+b x]^{4/3}} \end{aligned}$$

Result (type 5, 80 leaves):

$$\begin{aligned} & \left(3 \sin [a+b x]^{4/3}\right. \\ & \left.\left(2 \cos [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \cos [a+b x]^2\right]+\left(\sin [a+b x]^2\right)^{2/3}\right)\right) / \\ & \left(4 b \cos [a+b x]^{4/3}\left(\sin [a+b x]^2\right)^{2/3}\right) \end{aligned}$$

Problem 329: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[a + bx]^{1/3}}{\sin[a + bx]^{1/3}} dx$$

Optimal (type 3, 128 leaves, 8 steps) :

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}}{\sqrt{3}}\right]}{2 b}-\frac{\log \left[1+\frac{\cos [a+b x]^{4/3}}{\sin [a+b x]^{4/3}}-\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}\right]}{4 b}+\frac{\log \left[1+\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}\right]}{2 b}$$

Result (type 5, 57 leaves) :

$$-\left(\left(3 \cos [a+b x]^{4/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \cos [a+b x]^2\right] \sin [a+b x]^{2/3}\right)\right. \\ \left.\left(4 b (\sin [a+b x]^2)^{1/3}\right)\right)$$

Problem 330: Result unnecessarily involves higher level functions.

$$\int \frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}} dx$$

Optimal (type 3, 225 leaves, 11 steps) :

$$\frac{\operatorname{ArcTan}\left[\sqrt{3}-\frac{2 \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{2 b}-\frac{\operatorname{ArcTan}\left[\sqrt{3}+\frac{2 \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{2 b}-\frac{\operatorname{ArcTan}\left[\frac{\cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{b}- \\ \frac{\sqrt{3} \log \left[1+\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}-\frac{\sqrt{3} \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{4 b}+\frac{\sqrt{3} \log \left[1+\frac{\cos [a+b x]^{2/3}}{\sin [a+b x]^{2/3}}+\frac{\sqrt{3} \cos [a+b x]^{1/3}}{\sin [a+b x]^{1/3}}\right]}{4 b}$$

Result (type 5, 57 leaves) :

$$-\left(\left(3 \cos [a+b x]^{5/3} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \cos [a+b x]^2\right] \sin [a+b x]^{1/3}\right)\right. \\ \left.\left(5 b (\sin [a+b x]^2)^{1/6}\right)\right)$$

Problem 331: Result unnecessarily involves higher level functions.

$$\int \frac{\cos [a+b x]^{4/3}}{\sin [a+b x]^{4/3}} dx$$

Optimal (type 3, 250 leaves, 12 steps) :

$$\frac{\operatorname{ArcTan}\left[\sqrt{3}-\frac{2 \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}\right]}{2 b}-\frac{\operatorname{ArcTan}\left[\sqrt{3}+\frac{2 \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}\right]}{2 b}- \\ \frac{\operatorname{ArcTan}\left[\frac{\sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}\right]}{b}-\frac{\sqrt{3} \log \left[1-\frac{\sqrt{3} \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}+\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}\right]}{4 b}+ \\ \frac{\sqrt{3} \log \left[1+\frac{\sqrt{3} \sin [a+b x]^{1/3}}{\cos [a+b x]^{1/3}}+\frac{\sin [a+b x]^{2/3}}{\cos [a+b x]^{2/3}}\right]}{4 b}-\frac{3 \cos [a+b x]^{1/3}}{b \sin [a+b x]^{1/3}}$$

Result (type 5, 78 leaves) :

$$-\left(\left(3 \cos[a+b x]^{1/3} \right. \right. \\ \left. \left. - \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \cos[a+b x]^2\right] \sin[a+b x]^2 + (\sin[a+b x]^2)^{5/6} \right) \right) / \\ (b \sin[a+b x]^{1/3} (\sin[a+b x]^2)^{5/6}) \Big)$$

Problem 332: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[a+b x]^{5/3}}{\sin[a+b x]^{5/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \sin[a+b x]^{2/3}}{\cos[a+b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} + \frac{\operatorname{Log}\left[1+\frac{\sin[a+b x]^{2/3}}{\cos[a+b x]^{2/3}}\right]}{2 b} - \\ \frac{\operatorname{Log}\left[1-\frac{\sin[a+b x]^{2/3}}{\cos[a+b x]^{2/3}}+\frac{\sin[a+b x]^{4/3}}{\cos[a+b x]^{4/3}}\right]}{4 b} - \frac{3 \cos[a+b x]^{2/3}}{2 b \sin[a+b x]^{2/3}}$$

Result (type 5, 80 leaves):

$$-\left(\left(3 \cos[a+b x]^{2/3} \right. \right. \\ \left. \left. - \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \cos[a+b x]^2\right] \sin[a+b x]^2 + (\sin[a+b x]^2)^{2/3} \right) \right) / \\ (2 b \sin[a+b x]^{2/3} (\sin[a+b x]^2)^{2/3}) \Big)$$

Problem 333: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[a+b x]^{7/3}}{\sin[a+b x]^{7/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \cos[a+b x]^{2/3}}{\sin[a+b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} + \\ \frac{\operatorname{Log}\left[1+\frac{\cos[a+b x]^{4/3}}{\sin[a+b x]^{4/3}}-\frac{\cos[a+b x]^{2/3}}{\sin[a+b x]^{2/3}}\right]}{4 b} - \frac{\operatorname{Log}\left[1+\frac{\cos[a+b x]^{2/3}}{\sin[a+b x]^{2/3}}\right]}{2 b} - \frac{3 \cos[a+b x]^{4/3}}{4 b \sin[a+b x]^{4/3}}$$

Result (type 5, 80 leaves):

$$-\left(\left(3 \cos[a + bx]^{4/3} \right. \right. \\ \left. \left. - \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \cos[a + bx]^2\right] \sin[a + bx]^2 + (\sin[a + bx]^2)^{1/3} \right) \right) / \\ \left(4 b \sin[a + bx]^{4/3} (\sin[a + bx]^2)^{1/3} \right)$$

Problem 366: Result more than twice size of optimal antiderivative.

$$\int (d \cos[a + bx])^n (c \sin[a + bx])^{5/2} dx$$

Optimal (type 5, 76 leaves, 1 step):

$$-\left(\left(c (d \cos[a + bx])^{1+n} \text{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[a + bx]^2\right] (\sin[a + bx])^{3/2} \right) \right. \\ \left. \left(b d (1+n) (\sin[a + bx]^2)^{3/4} \right) \right)$$

Result (type 5, 158 leaves):

$$\left((d \cos[a + bx])^n \cot[a + bx] \left(- (3+n) \text{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[a + bx]^2\right] - \right. \right. \\ \left. \left. (3+n) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[a + bx]^2\right] + \right. \right. \\ \left. \left. (1+n) \cos[a + bx]^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3+n}{2}, \frac{5+n}{2}, \cos[a + bx]^2\right] \right) \right. \\ \left. \left(c \sin[a + bx] \right)^{5/2} \right) / \left(2 b (1+n) (3+n) (\sin[a + bx]^2)^{3/4} \right)$$

Problem 450: Result unnecessarily involves higher level functions.

$$\int \sqrt{b \sec[e + fx]} (a \sin[e + fx])^{9/2} dx$$

Optimal (type 3, 449 leaves, 13 steps):

$$\begin{aligned}
& - \frac{1}{32 \sqrt{2} \sqrt{b} f} 21 a^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]} + \\
& \frac{1}{32 \sqrt{2} \sqrt{b} f} 21 a^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]} + \\
& \frac{1}{64 \sqrt{2} \sqrt{b} f} 21 a^{9/2} \sqrt{b \cos[e+f x]} \\
& \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x]\right] \sqrt{b \sec[e+f x]} - \frac{1}{64 \sqrt{2} \sqrt{b} f} \\
& 21 a^{9/2} \sqrt{b \cos[e+f x]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x]\right] \sqrt{b \sec[e+f x]} - \\
& \frac{7 a^3 b (\sin[e+f x])^{3/2}}{16 f \sqrt{b \sec[e+f x]}} - \frac{a b (\sin[e+f x])^{7/2}}{4 f \sqrt{b \sec[e+f x]}}
\end{aligned}$$

Result (type 5, 109 leaves):

$$\begin{aligned}
& - \left(\left(a^4 \sqrt{b \sec[e+f x]} \sqrt{a \sin[e+f x]} \right. \right. \\
& \left. \left. \left(21 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+f x]^2\right] \sin[2(e+f x)] + \right. \right. \right. \\
& \left. \left. \left. (\sin[e+f x]^2)^{3/4} (9 \sin[2(e+f x)] - \sin[4(e+f x)]) \right) \right) \Big/ \left(32 f (\sin[e+f x]^2)^{3/4} \right)
\end{aligned}$$

Problem 451: Result unnecessarily involves higher level functions.

$$\int \sqrt{b \sec[e+f x]} (\sin[e+f x])^{5/2} dx$$

Optimal (type 3, 414 leaves, 12 steps):

$$\begin{aligned}
& - \frac{1}{4 \sqrt{2} \sqrt{b} f} 3 a^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]} + \\
& \frac{3 a^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{4 \sqrt{2} \sqrt{b} f} + \frac{1}{8 \sqrt{2} \sqrt{b} f} \\
& 3 a^{5/2} \sqrt{b \cos[e+f x]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x]\right] \sqrt{b \sec[e+f x]} - \\
& \frac{1}{8 \sqrt{2} \sqrt{b} f} 3 a^{5/2} \sqrt{b \cos[e+f x]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x]\right] \\
& \frac{\sqrt{b \sec[e+f x]}}{\sqrt{b \sec[e+f x]} - \frac{a b (\sin[e+f x])^{3/2}}{2 f \sqrt{b \sec[e+f x]}}}
\end{aligned}$$

Result (type 5, 87 leaves):

$$-\left(\left(a^2 \sqrt{b \sec [e+f x]} \sqrt{a \sin [e+f x]} \right.\right. \\ \left.\left(3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos [e+f x]^2\right] + (\sin [e+f x]^2)^{3/4}\right)\right. \\ \left.\left.\sin [2 (e+f x)]\right)\right) / \left(4 f (\sin [e+f x]^2)^{3/4}\right)$$

Problem 452: Result unnecessarily involves higher level functions.

$$\int \sqrt{b \sec [e+f x]} \sqrt{a \sin [e+f x]} dx$$

Optimal (type 3, 376 leaves, 11 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin [e+f x]}}{\sqrt{a} \sqrt{b \cos [e+f x]}}\right] \sqrt{b \cos [e+f x]} \sqrt{b \sec [e+f x]}}{\sqrt{2} \sqrt{b} f} + \\ \frac{\sqrt{a} \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin [e+f x]}}{\sqrt{a} \sqrt{b \cos [e+f x]}}\right] \sqrt{b \cos [e+f x]} \sqrt{b \sec [e+f x]}}{\sqrt{2} \sqrt{b} f} + \frac{1}{2 \sqrt{2} \sqrt{b} f} - \\ \frac{\sqrt{a} \sqrt{b \cos [e+f x]} \log \left[\sqrt{a}-\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin [e+f x]}}{\sqrt{b \cos [e+f x]}}+\sqrt{a} \tan [e+f x]\right] \sqrt{b \sec [e+f x]}}{\sqrt{2} \sqrt{b} \sqrt{b \cos [e+f x]}} - \\ \frac{1}{2 \sqrt{2} \sqrt{b} f} \\ \frac{\sqrt{a} \sqrt{b \cos [e+f x]} \log \left[\sqrt{a}+\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin [e+f x]}}{\sqrt{b \cos [e+f x]}}+\sqrt{a} \tan [e+f x]\right] \sqrt{b \sec [e+f x]}}{\sqrt{2} \sqrt{b} \sqrt{b \cos [e+f x]}}$$

Result (type 5, 67 leaves):

$$-\left(\left(\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos [e+f x]^2\right] \right.\right. \\ \left.\left.\sqrt{b \sec [e+f x]} \sqrt{a \sin [e+f x]} \sin [2 (e+f x)]\right)\right) / \left(f (\sin [e+f x]^2)^{3/4}\right)$$

Problem 456: Result unnecessarily involves higher level functions.

$$\int \sqrt{b \sec [e+f x]} (a \sin [e+f x])^{7/2} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$-\frac{5 a^3 b \sqrt{a \sin [e+f x]}}{6 f \sqrt{b \sec [e+f x]}} - \frac{a b (a \sin [e+f x])^{5/2}}{3 f \sqrt{b \sec [e+f x]}} + \\ \frac{5 a^4 \text{EllipticF}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{b \sec [e+f x]} \sqrt{\sin [2 e+2 f x]}}{12 f \sqrt{a \sin [e+f x]}}$$

Result (type 5, 90 leaves):

$$\left(a^3 b \sqrt{a \sin[e + f x]} \left(2 (-6 + \cos[2(e + f x)]) + 5 \csc[e + f x]^2 \right. \right. \\ \left. \left. \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{3/4} \right) \right) / \left(12 f \sqrt{b \sec[e + f x]} \right)$$

Problem 457: Result unnecessarily involves higher level functions.

$$\int \sqrt{b \sec[e + f x]} (a \sin[e + f x])^{3/2} dx$$

Optimal (type 4, 91 leaves, 4 steps):

$$- \frac{a b \sqrt{a \sin[e + f x]}}{f \sqrt{b \sec[e + f x]}} + \frac{a^2 \text{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e + f x]} \sqrt{\sin[2 e + 2 f x]}}{2 f \sqrt{a \sin[e + f x]}}$$

Result (type 5, 83 leaves):

$$\left(b \csc[e + f x]^3 (a \sin[e + f x])^{3/2} \right. \\ \left. \left(-1 + \cos[2(e + f x)] + \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{3/4} \right) \right) / \left(2 f \sqrt{b \sec[e + f x]} \right)$$

Problem 458: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b \sec[e + f x]}}{\sqrt{a \sin[e + f x]}} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$\frac{\text{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e + f x]} \sqrt{\sin[2 e + 2 f x]}}{f \sqrt{a \sin[e + f x]}}$$

Result (type 5, 66 leaves):

$$\left(\cot[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e + f x]^2\right] \sqrt{b \sec[e + f x]} (-\tan[e + f x]^2)^{3/4} \right) / \\ \left(f \sqrt{a \sin[e + f x]} \right)$$

Problem 459: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b \sec[e + f x]}}{(a \sin[e + f x])^{5/2}} dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$-\frac{2 b}{3 a f \sqrt{b \sec [e+f x]} (a \sin [e+f x])^{3/2}} + \frac{2 \text{EllipticF}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{b \sec [e+f x]} \sqrt{\sin [2 e+2 f x]}}{3 a^2 f \sqrt{a \sin [e+f x]}}$$

Result (type 5, 75 leaves):

$$\left(2 \cot [e+f x] \sqrt{b \sec [e+f x]} \left(-1 + \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec [e+f x]^2\right] (-\tan [e+f x]^2)^{3/4}\right)\right) / \left(3 a^2 f \sqrt{a \sin [e+f x]}\right)$$

Problem 460: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b \sec [e+f x]}}{(a \sin [e+f x])^{9/2}} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{2 b}{7 a f \sqrt{b \sec [e+f x]} (a \sin [e+f x])^{7/2}} - \frac{4 b}{7 a^3 f \sqrt{b \sec [e+f x]} (a \sin [e+f x])^{3/2}} + \frac{4 \text{EllipticF}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{b \sec [e+f x]} \sqrt{\sin [2 e+2 f x]}}{7 a^4 f \sqrt{a \sin [e+f x]}}$$

Result (type 5, 111 leaves):

$$-\left(\left(2 \cos [2 (e+f x)] (b \sec [e+f x])^{3/2} \left((-2+\cos [2 (e+f x)]) \csc [e+f x]^2 + 2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec [e+f x]^2\right] (-\tan [e+f x]^2)^{3/4}\right)\right) / \left(7 a^3 b f (-2+\sec [e+f x]^2) (a \sin [e+f x])^{3/2}\right)\right)$$

Problem 461: Result unnecessarily involves higher level functions.

$$\int \frac{\sin [e+f x]^{9/2}}{\sqrt{b \sec [e+f x]}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{7 b \sin [e+f x]^{3/2}}{30 f (b \sec [e+f x])^{3/2}} - \frac{b \sin [e+f x]^{7/2}}{5 f (b \sec [e+f x])^{3/2}} + \frac{7 \text{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{\sin [e+f x]}}{20 f \sqrt{b \sec [e+f x]} \sqrt{\sin [2 e+2 f x]}}$$

Result (type 5, 99 leaves):

$$\left(\sqrt{b \operatorname{Sec}[e + f x]} \left(4 (25 - 14 \cos[2(e + f x)] + 3 \cos[4(e + f x)]) \sin[e + f x]^2 - 84 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Sec}[e + f x]^2\right] (-\tan[e + f x]^2)^{1/4} \right) \right) / (480 b f \sqrt{\sin[e + f x]})$$

Problem 462: Result unnecessarily involves higher level functions.

$$\int \frac{\sin[e + f x]^{5/2}}{\sqrt{b \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{b \sin[e + f x]^{3/2}}{3 f (b \operatorname{Sec}[e + f x])^{3/2}} + \frac{\operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[e + f x]}}{2 f \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{\sin[2 e + 2 f x]}}$$

Result (type 5, 86 leaves):

$$\left(\sqrt{b \operatorname{Sec}[e + f x]} \left(5 - 6 \cos[2(e + f x)] + \cos[4(e + f x)] - 6 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Sec}[e + f x]^2\right] (-\tan[e + f x]^2)^{1/4} \right) \right) / (24 b f \sqrt{\sin[e + f x]})$$

Problem 463: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\sin[e + f x]}}{\sqrt{b \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 51 leaves, 3 steps):

$$\frac{\operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[e + f x]}}{f \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{\sin[2 e + 2 f x]}}$$

Result (type 5, 75 leaves):

$$-\left(\left(\sqrt{b \operatorname{Sec}[e + f x]} \left(-1 + \cos[2(e + f x)] + \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Sec}[e + f x]^2\right] (-\tan[e + f x]^2)^{1/4} \right) \right) / (2 b f \sqrt{\sin[e + f x]}) \right)$$

Problem 464: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{b \operatorname{Sec}[e + f x]} \sin[e + f x]^{3/2}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$-\frac{\frac{2 b}{f (b \operatorname{Sec}[e+f x])^{3/2} \sqrt{\sin[e+f x]}} - \frac{2 \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{\sin[e+f x]}}{f \sqrt{b \operatorname{Sec}[e+f x]} \sqrt{\sin[2 e+2 f x]}}$$

Result (type 5, 64 leaves):

$$\left(\sqrt{b \operatorname{Sec}[e+f x]} \left(-2 + \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Sec}[e+f x]^2\right] (-\operatorname{Tan}[e+f x]^2)^{1/4}\right)\right) / (b f \sqrt{\sin[e+f x]})$$

Problem 465: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{b \operatorname{Sec}[e+f x]} \sin[e+f x]^{7/2}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{\frac{2 b}{5 f (b \operatorname{Sec}[e+f x])^{3/2} \sin[e+f x]^{5/2}} - \frac{\frac{4 b}{5 f (b \operatorname{Sec}[e+f x])^{3/2} \sqrt{\sin[e+f x]}} - \frac{4 \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{\sin[e+f x]}}{5 f \sqrt{b \operatorname{Sec}[e+f x]} \sqrt{\sin[2 e+2 f x]}}}{5 b f \sin[e+f x]^{5/2}}$$

Result (type 5, 84 leaves):

$$\left(\sqrt{b \operatorname{Sec}[e+f x]} \left(-3 + \operatorname{Cos}[2 (e+f x)] + 2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Sec}[e+f x]^2\right] \sin[e+f x]^2 (-\operatorname{Tan}[e+f x]^2)^{1/4}\right)\right) / (5 b f \sin[e+f x]^{5/2})$$

Problem 466: Result unnecessarily involves higher level functions.

$$\int \frac{\sin[e+f x]^{3/2}}{\sqrt{b \operatorname{Sec}[e+f x]}} dx$$

Optimal (type 3, 363 leaves, 12 steps):

$$\begin{aligned} & \frac{\sqrt{b} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b \cos[e+f x]}}{\sqrt{b} \sqrt{\sin[e+f x]}}\right]}{4 \sqrt{2} f \sqrt{b \cos[e+f x]} \sqrt{b \operatorname{Sec}[e+f x]}} - \frac{\sqrt{b} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b \cos[e+f x]}}{\sqrt{b} \sqrt{\sin[e+f x]}}\right]}{4 \sqrt{2} f \sqrt{b \cos[e+f x]} \sqrt{b \operatorname{Sec}[e+f x]}} \\ & + \frac{\sqrt{b} \operatorname{Log}\left[\sqrt{b} + \sqrt{b} \operatorname{Cot}[e+f x] - \frac{\sqrt{2} \sqrt{b \cos[e+f x]}}{\sqrt{\sin[e+f x]}}\right]}{8 \sqrt{2} f \sqrt{b \cos[e+f x]} \sqrt{b \operatorname{Sec}[e+f x]}} \\ & - \frac{\sqrt{b} \operatorname{Log}\left[\sqrt{b} + \sqrt{b} \operatorname{Cot}[e+f x] + \frac{\sqrt{2} \sqrt{b \cos[e+f x]}}{\sqrt{\sin[e+f x]}}\right]}{8 \sqrt{2} f \sqrt{b \cos[e+f x]} \sqrt{b \operatorname{Sec}[e+f x]}} - \frac{b \sqrt{\sin[e+f x]}}{2 f (b \operatorname{Sec}[e+f x])^{3/2}} \end{aligned}$$

Result (type 5, 75 leaves):

$$-\left(\left(b \sqrt{\sin[e+f x]} \left(\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[e+f x]^2\right]+3 (\sin[e+f x]^2)^{1/4}\right)\right)\right)/\\ \left(6 f (b \sec[e+f x])^{3/2} (\sin[e+f x]^2)^{1/4}\right)$$

Problem 467: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{b \sec[e+f x]} \sqrt{\sin[e+f x]}} dx$$

Optimal (type 3, 328 leaves, 11 steps):

$$\frac{\frac{\sqrt{b} \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \cos[e+f x]}{\sqrt{b} \sqrt{\sin[e+f x]}}\right]}{\sqrt{b} \sqrt{\sin[e+f x]}}-\frac{\sqrt{b} \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \cos[e+f x]}{\sqrt{b} \sqrt{\sin[e+f x]}}\right]}{\sqrt{2} f \sqrt{b} \cos[e+f x] \sqrt{b} \sec[e+f x]}-\frac{\sqrt{2} f \sqrt{b} \cos[e+f x]}{\sqrt{2} f \sqrt{b} \cos[e+f x] \sqrt{b} \sec[e+f x]}-\frac{\sqrt{b} \log \left[\sqrt{b}+\sqrt{b} \cot[e+f x]-\frac{\sqrt{2} \sqrt{b} \cos[e+f x]}{\sqrt{\sin[e+f x]}}\right]}{2 \sqrt{2} f \sqrt{b} \cos[e+f x] \sqrt{b} \sec[e+f x]}+\frac{\sqrt{b} \log \left[\sqrt{b}+\sqrt{b} \cot[e+f x]+\frac{\sqrt{2} \sqrt{b} \cos[e+f x]}{\sqrt{\sin[e+f x]}}\right]}{2 \sqrt{2} f \sqrt{b} \cos[e+f x] \sqrt{b} \sec[e+f x]}}$$

Result (type 5, 60 leaves):

$$-\frac{2 b \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[e+f x]^2\right] \sqrt{\sin[e+f x]}}{3 f (b \sec[e+f x])^{3/2} (\sin[e+f x]^2)^{1/4}}$$

Problem 472: Result unnecessarily involves higher level functions.

$$\int \frac{(a \sin[e+f x])^{9/2}}{(b \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 490 leaves, 14 steps):

$$\begin{aligned}
& - \frac{1}{128 \sqrt{2} b^{5/2} f} 7 a^{9/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}} \right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]} + \\
& \frac{7 a^{9/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}} \right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{128 \sqrt{2} b^{5/2} f} + \frac{1}{256 \sqrt{2} b^{5/2} f} \\
& 7 a^{9/2} \sqrt{b \cos[e+f x]} \log \left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x] \right] \sqrt{b \sec[e+f x]} - \\
& \frac{1}{256 \sqrt{2} b^{5/2} f} 7 a^{9/2} \sqrt{b \cos[e+f x]} \log \left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x] \right] \\
& \frac{\sqrt{b \sec[e+f x]}}{192 b f \sqrt{b \sec[e+f x]}} - \frac{a (a \sin[e+f x])^{7/2}}{48 b f \sqrt{b \sec[e+f x]}} + \frac{(a \sin[e+f x])^{11/2}}{6 a b f \sqrt{b \sec[e+f x]}}
\end{aligned}$$

Result (type 5, 125 leaves) :

$$\begin{aligned}
& \left(a^4 \sec[e+f x]^2 \sqrt{a \sin[e+f x]} \right. \\
& \left(-21 \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+f x]^2 \right] \sin[2(e+f x)] + \right. \\
& \left. (\sin[e+f x]^2)^{3/4} (\sin[2(e+f x)] - 7 \sin[4(e+f x)] + 2 \sin[6(e+f x)]) \right) \Bigg) / \\
& (384 f (b \sec[e+f x])^{3/2} (\sin[e+f x]^2)^{3/4})
\end{aligned}$$

Problem 473: Result unnecessarily involves higher level functions.

$$\int \frac{(a \sin[e+f x])^{5/2}}{(b \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 453 leaves, 13 steps) :

$$\begin{aligned}
& - \frac{1}{32 \sqrt{2} b^{5/2} f} 3 a^{5/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}} \right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]} + \\
& \frac{3 a^{5/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}} \right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{32 \sqrt{2} b^{5/2} f} + \frac{1}{64 \sqrt{2} b^{5/2} f} \\
& 3 a^{5/2} \sqrt{b \cos[e+f x]} \log \left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x] \right] \sqrt{b \sec[e+f x]} - \\
& \frac{1}{64 \sqrt{2} b^{5/2} f} 3 a^{5/2} \sqrt{b \cos[e+f x]} \log \left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x] \right] \\
& \frac{\sqrt{b \sec[e+f x]}}{16 b f \sqrt{b \sec[e+f x]}} - \frac{a (a \sin[e+f x])^{3/2}}{4 a b f \sqrt{b \sec[e+f x]}} + \frac{(a \sin[e+f x])^{7/2}}{4 a b f \sqrt{b \sec[e+f x]}}
\end{aligned}$$

Result (type 5, 93 leaves) :

$$-\left(\left(a \left(a \sin[e+f x]\right)^{3/2} \left(3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+f x]^2\right] + (-1+2 \cos[2 (e+f x)]) (\sin[e+f x]^2)^{3/4}\right)\right) / \left(16 b f \sqrt{b \sec[e+f x]} (\sin[e+f x]^2)^{3/4}\right)\right)$$

Problem 474: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a \sin[e+f x]}}{(b \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 418 leaves, 12 steps):

$$\begin{aligned} & -\frac{\sqrt{a} \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{4 \sqrt{2} b^{5/2} f} + \\ & \frac{\sqrt{a} \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{4 \sqrt{2} b^{5/2} f} + \frac{1}{8 \sqrt{2} b^{5/2} f} - \\ & \frac{\sqrt{a} \sqrt{b \cos[e+f x]} \log \left[\sqrt{a}-\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}}+\sqrt{a} \tan[e+f x]\right] \sqrt{b \sec[e+f x]}}{8 \sqrt{2} b^{5/2} f} - \\ & \frac{1}{8 \sqrt{2} b^{5/2} f} \sqrt{a} \sqrt{b \cos[e+f x]} \log \left[\sqrt{a}+\frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}}+\sqrt{a} \tan[e+f x]\right] \\ & \frac{\sqrt{b \sec[e+f x]}}{2 a b f \sqrt{b \sec[e+f x]}} + \frac{(a \sin[e+f x])^{3/2}}{2 a b f \sqrt{b \sec[e+f x]}} \end{aligned}$$

Result (type 5, 82 leaves):

$$\left(\left(a \sin[e+f x]\right)^{3/2} \left(-\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+f x]^2\right] + (\sin[e+f x]^2)^{3/4}\right)\right) / \left(2 a b f \sqrt{b \sec[e+f x]} (\sin[e+f x]^2)^{3/4}\right)$$

Problem 475: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(b \sec[e+f x])^{3/2} (a \sin[e+f x])^{3/2}} dx$$

Optimal (type 3, 411 leaves, 12 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{\sqrt{2} a^{3/2} b^{5/2} f} - \\
& \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{a} \sqrt{b \cos[e+f x]}}\right] \sqrt{b \cos[e+f x]} \sqrt{b \sec[e+f x]}}{\sqrt{2} a^{3/2} b^{5/2} f} - \frac{1}{2 \sqrt{2} a^{3/2} b^{5/2} f} - \\
& \frac{\sqrt{b \cos[e+f x]} \log\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x]\right] \sqrt{b \sec[e+f x]}}{2 \sqrt{2} a^{3/2} b^{5/2} f} + \\
& \frac{1}{2 \sqrt{2} a^{3/2} b^{5/2} f} \sqrt{b \cos[e+f x]} \log\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+f x]}}{\sqrt{b \cos[e+f x]}} + \sqrt{a} \tan[e+f x]\right] \\
& \frac{\sqrt{b \sec[e+f x]}}{\sqrt{a} b f \sqrt{b \sec[e+f x]} \sqrt{a \sin[e+f x]}} - \frac{2}{\sqrt{a} b f \sqrt{b \sec[e+f x]} \sqrt{a \sin[e+f x]}}
\end{aligned}$$

Result (type 5, 89 leaves):

$$\begin{aligned}
& \left(2 \left(\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+f x]^2\right] \sin[e+f x]^2 - (\sin[e+f x]^2)^{3/4}\right)\right) / \\
& (a b f \sqrt{b \sec[e+f x]} \sqrt{a \sin[e+f x]} (\sin[e+f x]^2)^{3/4})
\end{aligned}$$

Problem 477: Result unnecessarily involves higher level functions.

$$\int \frac{(a \sin[e+f x])^{7/2}}{(b \sec[e+f x])^{3/2}} dx$$

Optimal (type 4, 172 leaves, 6 steps):

$$\begin{aligned}
& -\frac{a^3 \sqrt{a \sin[e+f x]}}{12 b f \sqrt{b \sec[e+f x]}} - \frac{a (a \sin[e+f x])^{5/2}}{30 b f \sqrt{b \sec[e+f x]}} + \frac{(a \sin[e+f x])^{9/2}}{5 a b f \sqrt{b \sec[e+f x]}} + \\
& \frac{a^4 \text{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e + 2 f x]}}{24 b^2 f \sqrt{a \sin[e+f x]}}
\end{aligned}$$

Result (type 5, 103 leaves):

$$\begin{aligned}
& -\left(\left(a^5 \left(-4 + 17 \cos[2 (e+f x)] - 16 \cos[4 (e+f x)] + 3 \cos[6 (e+f x)] - \right.\right.\right. \\
& \left.\left.\left.20 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+f x]^2\right] (-\tan[e+f x]^2)^{3/4}\right)\right) / \\
& (480 b f \sqrt{b \sec[e+f x]} (a \sin[e+f x])^{3/2})
\end{aligned}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \frac{(a \sin[e+f x])^{3/2}}{(b \sec[e+f x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$-\frac{a \sqrt{a \sin[e+f x]}}{6 b f \sqrt{b \sec[e+f x]}} + \frac{(a \sin[e+f x])^{5/2}}{3 a b f \sqrt{b \sec[e+f x]}} +$$

$$\frac{a^2 \text{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e+2 f x]}}{12 b^2 f \sqrt{a \sin[e+f x]}}$$

Result (type 5, 87 leaves):

$$\left(a \sqrt{a \sin[e+f x]} \left(-2 \cos[2(e+f x)] + \csc[e+f x]^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+f x]^2\right] (-\tan[e+f x]^2)^{3/4} \right) \right) / (12 b f \sqrt{b \sec[e+f x]})$$

Problem 479: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(b \sec[e+f x])^{3/2} \sqrt{a \sin[e+f x]}} dx$$

Optimal (type 4, 94 leaves, 4 steps):

$$-\frac{\sqrt{a \sin[e+f x]}}{a b f \sqrt{b \sec[e+f x]}} + \frac{\text{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e+2 f x]}}{2 b^2 f \sqrt{a \sin[e+f x]}}$$

Result (type 5, 84 leaves):

$$-\left(\left(\cot[e+f x] \sqrt{b \sec[e+f x]} \left(-1 + \cos[2(e+f x)] - \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+f x]^2\right] (-\tan[e+f x]^2)^{3/4} \right) \right) / (2 b^2 f \sqrt{a \sin[e+f x]}) \right)$$

Problem 480: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(b \sec[e+f x])^{3/2} (a \sin[e+f x])^{5/2}} dx$$

Optimal (type 4, 100 leaves, 4 steps):

$$-\frac{2}{3 a b f \sqrt{b \sec[e+f x]} (a \sin[e+f x])^{3/2}} -$$

$$\frac{\text{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e+2 f x]}}{3 a^2 b^2 f \sqrt{a \sin[e+f x]}}$$

Result (type 5, 78 leaves):

$$-\left(\left(\operatorname{Cot}[e+f x] \sqrt{b \operatorname{Sec}[e+f x]} \right.\right. \\ \left.\left(2+\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e+f x]^2\right] (-\operatorname{Tan}[e+f x]^2)^{3/4}\right)\right) \Big/ \left(3 a^2 b^2 f \sqrt{a \operatorname{Sin}[e+f x]}\right)$$

Problem 481: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(b \operatorname{Sec}[e+f x])^{3/2} (a \operatorname{Sin}[e+f x])^{9/2}} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$-\frac{2}{7 a b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{7/2}} + \frac{2}{21 a^3 b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{3/2}} - \\ \frac{2 \operatorname{EllipticF}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{b \operatorname{Sec}[e+f x]} \sqrt{\operatorname{Sin}[2 e+2 f x]}}{21 a^4 b^2 f \sqrt{a \operatorname{Sin}[e+f x]}}$$

Result (type 5, 119 leaves):

$$\left(\operatorname{Cos}[2 (e+f x)] \operatorname{Csc}[e+f x]^4 \sqrt{a \operatorname{Sin}[e+f x]} \left((5+\operatorname{Cos}[2 (e+f x)]) \operatorname{Sec}[e+f x]^2 - \right.\right. \\ \left.\left.2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e+f x]^2\right] (-\operatorname{Tan}[e+f x]^2)^{7/4}\right)\right) \Big/ \left(21 a^5 b f \sqrt{b \operatorname{Sec}[e+f x]} (-2+\operatorname{Sec}[e+f x]^2)\right)$$

Problem 482: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(b \operatorname{Sec}[e+f x])^{3/2} (a \operatorname{Sin}[e+f x])^{13/2}} dx$$

Optimal (type 4, 174 leaves, 6 steps):

$$-\frac{2}{11 a b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{11/2}} + \\ \frac{2}{77 a^3 b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{7/2}} + \frac{4}{77 a^5 b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{3/2}} - \\ \frac{4 \operatorname{EllipticF}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{b \operatorname{Sec}[e+f x]} \sqrt{\operatorname{Sin}[2 e+2 f x]}}{77 a^6 b^2 f \sqrt{a \operatorname{Sin}[e+f x]}}$$

Result (type 5, 131 leaves):

$$\begin{aligned} & \left(2 \operatorname{Cot}[2(e + fx)] \operatorname{Csc}[2(e + fx)] \right. \\ & \quad \left. \sqrt{a \sin[e + fx]} \left((23 + 6 \cos[2(e + fx)] - \cos[4(e + fx)]) \operatorname{Csc}[e + fx]^4 + \right. \right. \\ & \quad \left. \left. 8 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e + fx]^2\right] (-\operatorname{Tan}[e + fx]^2)^{3/4} \right) \right) / \\ & \quad \left(77 a^7 b f \sqrt{b \operatorname{Sec}[e + fx]} (-2 + \operatorname{Sec}[e + fx]^2) \right) \end{aligned}$$

Problem 488: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[e + fx]^n \sin[e + fx]^m dx$$

Optimal (type 5, 86 leaves, 2 steps):

$$\begin{aligned} & -\frac{1}{f(1-n)} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+fx]^2\right] \\ & \quad \operatorname{Sec}[e+fx]^{-1+n} \sin[e+fx]^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}} \end{aligned}$$

Result (type 6, 2938 leaves):

$$\begin{aligned} & \left(2 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\ & \quad \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \operatorname{Sec}[e+fx]^n \\ & \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \sin[e+fx]^{2m} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) / \\ & \quad \left(f(1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\ & \quad n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\ & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\ & \quad \left(\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\ & \quad \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \sin[e+fx]^m \right) / \\ & \quad \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2] - 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\ & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \\ & \quad n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \end{aligned}$$

$$\begin{aligned}
& \left(2m(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \right. \\
& \quad \cos[e+f x] \left(\sec \left[\frac{1}{2}(e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2}(e+f x) \right]^2 \sec[e+f x] \right)^n \\
& \quad \sin[e+f x]^{-1+m} \tan \left[\frac{1}{2}(e+f x) \right] \Big/ \left((1+m) \right. \\
& \quad \left. \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] - \right. \right. \\
& \quad 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(e+f x) \right]^2 \Big) + \\
& \quad \left(2(3+m)(-1+n) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \left(\sec \left[\frac{1}{2}(e+f x) \right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2}(e+f x) \right]^2 \sec[e+f x] \right)^n \sin[e+f x]^m \tan \left[\frac{1}{2}(e+f x) \right]^2 \right) \Big/ \\
& \quad \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(e+f x) \right]^2 \Big) + \\
& \quad \left(2(3+m) \left(\sec \left[\frac{1}{2}(e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2}(e+f x) \right]^2 \sec[e+f x] \right)^n \sin[e+f x]^m \right. \\
& \quad \left. \tan \left[\frac{1}{2}(e+f x) \right] \left(-\frac{1}{3+m}(1+m)(1+m-n) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, n, 2+m-n, 1+\frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \sec \left[\frac{1}{2}(e+f x) \right]^2 \tan \left[\frac{1}{2}(e+f x) \right] + \right. \\
& \quad \left. \frac{1}{3+m}(1+m)n \operatorname{AppellF1} \left[1+\frac{1+m}{2}, 1+n, 1+m-n, 1+\frac{3+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \sec \left[\frac{1}{2}(e+f x) \right]^2 \tan \left[\frac{1}{2}(e+f x) \right] \right) \Big) \Big/ \\
& \quad \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2}(e+f x) \right]^2, -\tan \left[\frac{1}{2}(e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(e+f x) \right]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \sin [e+f x]^m \tan \left[\frac{1}{2} (e+f x) \right] \\
& \quad \left(-2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] -n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right. \\
& \quad \left(3+m \right) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, n, 2+m-n, 1+\frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right. \\
& \quad \left. \frac{1}{3+m} (1+m) n \operatorname{AppellF1} \left[1+\frac{1+m}{2}, 1+n, 1+m-n, 1+\frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \right) -2 \tan \left[\frac{1}{2} (e+f x) \right]^2 \\
& \quad \left((1+m-n) \left(-\frac{1}{5+m} (3+m) (2+m-n) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, n, 3+m-n, 1+\frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right. \\
& \quad \left. \frac{1}{5+m} (3+m) n \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 1+n, 2+m-n, 1+\frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \right) - \\
& \quad n \left(-\frac{1}{5+m} (3+m) (1+m-n) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 1+n, 2+m-n, 1+\frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right. \\
& \quad \left. \frac{1}{5+m} (3+m) (1+n) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 2+n, 1+m-n, 1+\frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \\
& \quad \Bigg((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] -2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] -n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \Bigg)^2 \Bigg) + \\
& \quad \left(2 (3+m) n \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left. \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^{-1+n} \sin [e+f x]^m \right)
\end{aligned}$$

$$\begin{aligned} & \text{Tan}\left[\frac{1}{2}(e+f x)\right] \left(-\cos\left[\frac{1}{2}(e+f x)\right] \sec[e+f x] \sin\left[\frac{1}{2}(e+f x)\right] + \right. \\ & \left. \cos\left[\frac{1}{2}(e+f x)\right]^2 \sec[e+f x] \tan[e+f x] \right) / \\ & \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \\ & \left. \left. \left. \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] - 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \right. \\ & \left. \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] - n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \\ & \left. \left. \left. \left. n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+f x)\right]^2 \right) \right) \end{aligned}$$

Problem 489: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sec[e+f x]^n (a \sin[e+f x])^m dx$$

Optimal (type 5, 89 leaves, 2 steps):

$$\begin{aligned} & -\frac{1}{f(1-n)} a \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+f x]^2\right] \\ & \sec[e+f x]^{-1+n} (a \sin[e+f x])^{-1+m} (\sin[e+f x]^2)^{\frac{1-m}{2}} \end{aligned}$$

Result (type 6, 2946 leaves):

$$\begin{aligned} & \left(2(3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] \right. \\ & \left(\sec\left[\frac{1}{2}(e+f x)\right]^2 \right)^{-1+n} \sec[e+f x]^n \left(\cos\left[\frac{1}{2}(e+f x)\right]^2 \sec[e+f x] \right)^n \\ & \left. \sin[e+f x]^m (a \sin[e+f x])^m \tan\left[\frac{1}{2}(e+f x)\right] \right) / \\ & \left(f(1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] - \right. \right. \\ & 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] - \right. \\ & n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \\ & \left. \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+f x)\right]^2 \right) \\ & \left(\left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2 \right] \right. \right. \\ & \left(\sec\left[\frac{1}{2}(e+f x)\right]^2 \right)^n \left(\cos\left[\frac{1}{2}(e+f x)\right]^2 \sec[e+f x] \right)^n \sin[e+f x]^m \right) / \\ & \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2] - 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right.\right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right.\right. \\
& \left. \left. n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) + \\
& \left(2m(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \left. \cos[e+fx] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \right. \\
& \left. \sin[e+fx]^{-1+m} \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big/ \left((1+m) \right. \\
& \left. \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(2(3+m)(-1+n) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\
& \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big/ \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \\
& \left. \left. \left. n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(2(3+m) \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \right. \\
& \left. \tan\left[\frac{1}{2}(e+fx)\right] \left(-\frac{1}{3+m}(1+m)(1+m-n) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, n, 2+m-n, 1+\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{1}{3+m}(1+m)n \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1+n, 1+m-n, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2] - 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \\
& \left. \left. n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left(2(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \left. \left(-2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. (3+m) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, n, 2+m-n, 1+\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{1}{3+m} (1+m) n \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1+n, 1+m-n, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left((1+m-n) \left(-\frac{1}{5+m} (3+m) (2+m-n) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, n, 3+m-n, 1+\frac{5+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{1}{5+m} (3+m) n \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+n, 2+m-n, 1+\frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - \\
& n \left(-\frac{1}{5+m} (3+m) (1+m-n) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+n, 2+m-n, 1+\frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{1}{5+m} (3+m) (1+n) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 2+n, 1+m-n, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big)^2 + \\
& \left(2(3+m)n \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+n} \sin[e+fx]^m \right. \\
& \left. \tan\left[\frac{1}{2}(e+fx)\right] \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right) / \\
& \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \\
& \left. \left. \left. n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)
\end{aligned}$$

Problem 490: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \sec[e+fx])^n \sin[e+fx]^m dx$$

Optimal (type 5, 89 leaves, 2 steps):

$$\begin{aligned}
& -\frac{1}{f(1-n)} b \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+fx]^2\right] \\
& (b \sec[e+fx])^{-1+n} \sin[e+fx]^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}}
\end{aligned}$$

Result (type 6, 2940 leaves):

$$\begin{aligned}
& \left(2(3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} (b \sec[e+fx])^n \right. \\
& \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^{2m} \tan\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left(f(1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \left. \left. n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \left(\sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right)^n \left(\cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \sec [\mathbf{e} + \mathbf{f} x] \right)^n \sin [\mathbf{e} + \mathbf{f} x]^m \right) \right) / \\
& \quad \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right) + \\
& \quad \left(2m (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right. \\
& \quad \left. \cos [\mathbf{e} + \mathbf{f} x] \left(\sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \sec [\mathbf{e} + \mathbf{f} x] \right)^n \right. \\
& \quad \left. \sin [\mathbf{e} + \mathbf{f} x]^{-1+m} \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \right) / \left((1+m) \right. \\
& \quad \left. \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right) + \\
& \quad \left(2 (3+m) (-1+n) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \left(\sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \sec [\mathbf{e} + \mathbf{f} x] \right)^n \sin [\mathbf{e} + \mathbf{f} x]^m \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right) / \\
& \quad \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right) + \\
& \quad \left(2 (3+m) \left(\sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \sec [\mathbf{e} + \mathbf{f} x] \right)^n \sin [\mathbf{e} + \mathbf{f} x]^m \right. \\
& \quad \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \right) +
\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3+m} (1+m) n \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \\
& \quad \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\Bigg) \Bigg) \Bigg/ \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \right. \right. \\
& \quad \left. \left. \left. n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \tan\left[\frac{1}{2}(e+f x)\right]^2 \right) \Bigg) - \\
& \left(2 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \\
& \quad \left(\sec\left[\frac{1}{2}(e+f x)\right]^2 \right)^{-1+n} \left(\cos\left[\frac{1}{2}(e+f x)\right]^2 \sec[e+f x] \right)^n \sin[e+f x]^m \tan\left[\frac{1}{2}(e+f x)\right] \\
& \quad \left(-2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. (3+m) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. \frac{1}{3+m} (1+m) n \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] \right) - 2 \tan\left[\frac{1}{2}(e+f x)\right]^2 \\
& \quad \left((1+m-n) \left(-\frac{1}{5+m} (3+m) (2+m-n) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, n, 3+m-n, 1 + \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. \frac{1}{5+m} (3+m) n \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] \right) - \\
& \quad n \left(-\frac{1}{5+m} (3+m) (1+m-n) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. \frac{1}{5+m} (3+m) (1+n) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 2+n, 1+m-n, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left((1+m) \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& - \tan \left[\frac{1}{2} (e+f x) \right]^2] - 2 \left((1+m-n) \text{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\
& \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2] - n \text{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \\
& \left. \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^2 + \\
& \left(2 (3+m) n \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^{-1+n} \sin [e+f x]^m \\
& \tan \left[\frac{1}{2} (e+f x) \right] \left(-\cos \left[\frac{1}{2} (e+f x) \right] \sec [e+f x] \sin \left[\frac{1}{2} (e+f x) \right] + \right. \\
& \left. \left. \cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \tan [e+f x] \right) \right) / \\
& \left((1+m) \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& - \tan \left[\frac{1}{2} (e+f x) \right]^2] - 2 \left((1+m-n) \text{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \\
& \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2] - n \text{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \\
& \left. \left. \left. n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right)
\end{aligned}$$

Problem 491: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \sec [e+f x])^n (a \sin [e+f x])^m dx$$

Optimal (type 5, 92 leaves, 2 steps):

$$\begin{aligned}
& -\frac{1}{f (1-n)} a b \text{Hypergeometric2F1} \left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos [e+f x]^2 \right] \\
& (b \sec [e+f x])^{-1+n} (a \sin [e+f x])^{-1+m} (\sin [e+f x]^2)^{\frac{1-m}{2}}
\end{aligned}$$

Result (type 6, 2948 leaves):

$$\begin{aligned}
& \left(2 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} (b \sec [e+f x])^n \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \\
& \sin [e+f x]^m (a \sin [e+f x])^m \tan \left[\frac{1}{2} (e+f x) \right] \Big) / \\
& \left(f (1+m) \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\left(1 + m - n \right) \text{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - \right. \\
& \quad n \text{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \\
& \left(\left(3+m \right) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^n \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \sin [e+f x]^m \Big) / \\
& \left(\left(1+m \right) \left(\left(3+m \right) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - 2 \left(\left(1+m-n \right) \text{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - n \text{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \Big) + \\
& \left(2m \left(3+m \right) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \cos [e+f x] \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \\
& \quad \left. \sin [e+f x]^{-1+m} \tan \left[\frac{1}{2} (e+f x) \right] \right) / \left(\left(1+m \right) \right. \\
& \quad \left(\left(3+m \right) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(\left(1+m-n \right) \text{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - n \text{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \Big) + \\
& \left(2 \left(3+m \right) \left(-1+n \right) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \sin [e+f x]^m \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) / \\
& \left(\left(1+m \right) \left(\left(3+m \right) \text{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - 2 \left(\left(1+m-n \right) \text{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - n \text{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \Big) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 (3+m) \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \sin [e+f x]^m \right. \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right] \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2} \right], \right. \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \\
& \quad \left. \frac{1}{3+m} (1+m) n \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2} \right], \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \right) \Bigg) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2} \right], \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2} \right], \right. \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \left. \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2} \right], \\
& \quad \left. \left. n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \Bigg) - \\
& \left(2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2} \right], \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \\
& \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^n \sin [e+f x]^m \tan \left[\frac{1}{2} (e+f x) \right] \\
& \left(-2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2} \right], \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2} \right], \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \left. \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \\
& \quad (3+m) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2} \right], \right. \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \\
& \quad \left. \frac{1}{3+m} (1+m) n \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2} \right], \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \\
& \quad \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \Bigg) - 2 \tan \left[\frac{1}{2} (e+f x) \right]^2 \\
& \left((1+m-n) \left(-\frac{1}{5+m} (3+m) (2+m-n) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, n, 3+m-n, 1 + \frac{5+m}{2} \right], \right. \right. \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \\
& \quad \left. \frac{1}{5+m} (3+m) n \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2} \right], \right. \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \Bigg) - \\
& \left. n \left(-\frac{1}{5+m} (3+m) (1+m-n) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2} \right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\left(1+m \right) \left(3+m \right) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right. \right. \\
& \left. \left. \frac{1}{5+m} (3+m) (1+n) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 2+n, 1+m-n, 1+\frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \right] \right) \right) / \\
& \left(\left(1+m \right) \left(3+m \right) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) + \\
& \left(2 (3+m) n \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^{-1+n} \sin [e+f x]^m \right. \\
& \left. \tan \left[\frac{1}{2} (e+f x) \right] \left(-\cos \left[\frac{1}{2} (e+f x) \right] \sec [e+f x] \sin \left[\frac{1}{2} (e+f x) \right] + \cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \tan [e+f x] \right) \right) / \\
& \left(\left(1+m \right) \left(3+m \right) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \right)
\end{aligned}$$

Problem 495: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc [e+f x] (b \sec [e+f x])^n dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1} \left[1, \frac{1+n}{2}, \frac{3+n}{2}, \sec [e+f x]^2 \right] (b \sec [e+f x])^{1+n}}{b f (1+n)}$$

Result (type 6, 2658 leaves):

$$\begin{aligned}
& \left((-2+n) \right. \\
& \left. \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{1}{2} \cos [e+f x] \sec \left[\frac{1}{2} (e+f x) \right]^2, \cos [e+f x] \sec \left[\frac{1}{2} (e+f x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right)^2 \csc[\mathbf{e} + \mathbf{f} x] \sec[\mathbf{e} + \mathbf{f} x]^{-1+n} (b \sec[\mathbf{e} + \mathbf{f} x])^n \right) / \\
& \left(f (-1+n) \left(2 (-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \\
& \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cos[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 + \right. \right. \\
& \left. \left. n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \\
& \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] - 2 \text{AppellF1}[2-n, -n, 2, 3-n, \right. \right. \\
& \left. \left. \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) \cos[\mathbf{e} + \mathbf{f} x] \right) \\
& \left(- \left(\left((-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \right. \\
& \left. \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)] \csc[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \sec[\mathbf{e} + \mathbf{f} x]^{-1+n} \right) / \right. \right. \\
& \left. \left. \left((-1+n) \left(2 (-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \right. \\
& \left. \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cos[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 + \right. \right. \right. \\
& \left. \left. \left. n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \right. \\
& \left. \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] - 2 \text{AppellF1}[2-n, -n, 2, 3-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \right. \right. \right. \\
& \left. \left. \left. \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) \cos[\mathbf{e} + \mathbf{f} x] \right) \right) + \\
& \left((-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \\
& \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \sec[\mathbf{e} + \mathbf{f} x]^n \sin[\mathbf{e} + \mathbf{f} x] \right) / \right. \right. \\
& \left. \left(2 (-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \\
& \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cos[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 + \right. \right. \\
& \left. \left. n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \right. \right. \\
& \left. \left. \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] - 2 \text{AppellF1}[2-n, -n, 2, 3-n, \right. \right. \\
& \left. \left. \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) \cos[\mathbf{e} + \mathbf{f} x] \right) + \\
& \left((-2+n) \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \sec[\mathbf{e} + \mathbf{f} x]^{-1+n} \left(- \frac{1}{2-n} (1-n) n \text{AppellF1}[2-n, 1-n, \right. \right. \\
& \left. \left. 1, 3-n, \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right. \right. \\
& \left. \left. - \frac{1}{2} \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \sin[\mathbf{e} + \mathbf{f} x] + \frac{1}{2} \cos[\mathbf{e} + \mathbf{f} x] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right. \right)
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{\tan\left(\frac{1}{2}(e+fx)\right)}{2-n} + \frac{1}{2-n}(1-n) \operatorname{AppellF1}\left[2-n, -n, 2, \right. \right. \\
 & \left. \left. 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
 & \left. \left(-\sec\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
 & \left. \left((-1+n) \left(2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \cos\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \right. \\
 & \left. \left. \left. \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \right. \right. \right. \\
 & \left. \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \cos[e+fx] \right) \right) - \right. \\
 & \left. \left((-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]^{-1+n} \right. \right. \\
 & \left. \left. \left(-2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \cos\left[\frac{1}{2}(e+fx)\right] \sin\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\
 & \left. \left. \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \right. \right. \right. \right. \\
 & \left. \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \right. \right. \right. \\
 & \left. \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \sin[e+fx] + 2(-2+n) \right. \right. \\
 & \left. \left. \cos\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{1}{2-n}(1-n) n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \right. \right. \right. \right. \\
 & \left. \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \left(-\frac{1}{2} \sec\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
 & \left. \left. \left. \sin[e+fx] + \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \frac{1}{2-n} \right. \right. \\
 & \left. \left. (1-n) \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \left(-\sec\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \right. \right. \\
 & \left. \left. \left. \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \right. \right. \\
 & \left. \left. \cos[e+fx] \left(n \left(\frac{1}{3-n}(1-n)(2-n) \operatorname{AppellF1}\left[3-n, 2-n, 1, 4-n, \frac{1}{2} \cos[e+fx] \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right] \left(-\frac{1}{2} \sec\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
 & \left. \left. \left. \right. \right. \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2 \tan[\frac{1}{2}(e + fx)] \right) + \frac{1}{3-n} \right. \\
& (2-n) \operatorname{AppellF1}[3-n, 1-n, 2, 4-n, \frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2, \\
& \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2] \left(-\sec[\frac{1}{2}(e + fx)]^2 \sin[e + fx] + \right. \\
& \left. \left. \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2 \tan[\frac{1}{2}(e + fx)] \right) \right) - \\
& 2 \left(-\frac{1}{3-n} (2-n) n \operatorname{AppellF1}[3-n, 1-n, 2, 4-n, \frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2, \right. \\
& \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2] \left(-\frac{1}{2} \sec[\frac{1}{2}(e + fx)]^2 \sin[e + fx] + \right. \\
& \left. \left. \frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2 \tan[\frac{1}{2}(e + fx)] \right) \right) + \frac{1}{3-n} \\
& 2 (2-n) \operatorname{AppellF1}[3-n, -n, 3, 4-n, \frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2, \\
& \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2] \left(-\sec[\frac{1}{2}(e + fx)]^2 \sin[e + fx] + \right. \\
& \left. \left. \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2 \tan[\frac{1}{2}(e + fx)] \right) \right) \Bigg) \Bigg) / \\
& \left((-1+n) \left(2 (-2+n) \operatorname{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2, \right. \right. \\
& \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2] \cos[\frac{1}{2}(e + fx)]^2 + \\
& \left. \left. \left(n \operatorname{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2, \right. \right. \right. \\
& \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2] - 2 \operatorname{AppellF1}[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e + fx] \\
& \left. \left. \left. \sec[\frac{1}{2}(e + fx)]^2, \cos[e + fx] \sec[\frac{1}{2}(e + fx)]^2] \right) \cos[e + fx] \right)^2 \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 496: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc[e + fx]^3 (b \sec[e + fx])^n dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1}[2, \frac{3+n}{2}, \frac{5+n}{2}, \sec[e + fx]^2] (b \sec[e + fx])^{3+n}}{b^3 f (3+n)}$$

Result (type 6, 5198 leaves):

$$\left(\csc[e + fx]^3 (b \sec[e + fx])^n \left(\frac{1 + \tan[\frac{1}{2}(e + fx)]^2}{1 - \tan[\frac{1}{2}(e + fx)]^2} \right)^n \right)$$

$$\begin{aligned}
& \left(- \left(\text{AppellF1}[1, n, -n, 2, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \right) \right. \\
& \quad \left(n \left(\text{AppellF1}[2, n, 1-n, 3, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] + \right. \right. \\
& \quad \quad \left. \text{AppellF1}[2, 1+n, -n, 3, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \right) + \\
& \quad \left. 2 \text{AppellF1}[1, n, -n, 2, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) + \\
& \quad \left(\text{AppellF1}[1, n, -n, 2, \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) \right) / \\
& \quad \left(2 \text{AppellF1}[1, n, -n, 2, \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] + \right. \\
& \quad \quad n \left(\text{AppellF1}[2, n, 1-n, 3, \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] + \text{AppellF1}[2, \right. \\
& \quad \quad \quad \left. 1+n, -n, 3, \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \right) \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) + \\
& \quad \left(2 (-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \left(1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right), 1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \right. \\
& \quad \quad \left. \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \left(-1 + \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) \right) / \\
& \quad \left((-1+n) \left(-2 (-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} \left(1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right), \right. \right. \\
& \quad \quad \left. \left. 1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right] + \left(n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2} \left(1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right), \right. \right. \\
& \quad \quad \quad \left. \left. 1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right] - 2 \text{AppellF1}[2-n, -n, 2, 3-n, \frac{1}{2} \right. \\
& \quad \quad \quad \left. \left. \left(1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right), 1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) \left(-1 + \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) \right) \right) \right) / \\
& 4 f \left(\frac{1}{4} n \left(\frac{1 + \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2}{1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2} \right)^{-1+n} \left(\frac{\sec[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]}{1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2} + \right. \right. \\
& \quad \left. \frac{\sec[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)] \left(1 + \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right)}{\left(1 - \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right)^2} \right) \\
& \quad \left(- \left(\text{AppellF1}[1, n, -n, 2, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \right) \right. \\
& \quad \quad \left(n \left(\text{AppellF1}[2, n, 1-n, 3, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] + \right. \right. \\
& \quad \quad \quad \left. \text{AppellF1}[2, 1+n, -n, 3, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \right) + 2 \text{AppellF1}[\right. \\
& \quad \quad \quad \left. 1, n, -n, 2, \cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) + \\
& \quad \quad \quad \left(\text{AppellF1}[1, n, -n, 2, \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2] \tan[\frac{1}{2}(\mathbf{e} + \mathbf{f} x)]^2 \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{AppellF1}[1, n, -n, 2, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] + \right. \\
& \quad n \left(\operatorname{AppellF1}[2, n, 1-n, 3, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] + \operatorname{AppellF1}[2, 1+n, -n, 3, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) + \\
& \left(2 (-2+n) \operatorname{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} (1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2), \right. \\
& \quad \left. 1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \left(-1+\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) \right) / \\
& \left((-1+n) \left(-2 (-2+n) \operatorname{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2} (1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2), \right. \right. \\
& \quad \left. \left. 1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] + \left(n \operatorname{AppellF1}[2-n, 1-n, 1, 3-n, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{2} (1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2), 1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] - 2 \operatorname{AppellF1}[2-n, \right. \right. \right. \\
& \quad \left. \left. \left. -n, 2, 3-n, \frac{1}{2} (1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2), 1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) \right) \right) / \\
& \left(-1+\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) \Big) \Big) + \frac{1}{4} \left(\frac{1+\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2}{1-\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2} \right)^n \\
& \left(-\frac{1}{2} n \operatorname{AppellF1}[2, n, 1-n, 3, \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right. \\
& \quad \left. \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)] \csc[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 - \frac{1}{2} n \operatorname{AppellF1}[2, 1+n, -n, 3, \right. \\
& \quad \left. \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)] \csc[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) / \\
& \left(n \left(\operatorname{AppellF1}[2, n, 1-n, 3, \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] + \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}[2, 1+n, -n, 3, \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) + 2 \operatorname{AppellF1}[\right. \\
& \quad \left. 1, n, -n, 2, \cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\cot[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) \Big) + \\
& \left(\operatorname{AppellF1}[1, n, -n, 2, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right. \\
& \quad \left. \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)] \right) / \left(2 \operatorname{AppellF1}[1, n, -n, 2, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] + \right. \\
& \quad n \left(\operatorname{AppellF1}[2, n, 1-n, 3, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] + \operatorname{AppellF1}[2, 1+n, \right. \\
& \quad \left. \left. -n, 3, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right) \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) + \\
& \left(\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \left(\frac{1}{2} n \operatorname{AppellF1}[2, n, 1-n, 3, \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2, -\tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2] \right. \right. \\
& \quad \left. \left. \sec[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)] + \frac{1}{2} n \operatorname{AppellF1}[2, 1+n, -n, 3, \right. \right. \\
& \quad \left. \left. \tan[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)]^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \\
& n \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{2}{3}(1-n) \text{AppellF1}[3, n, 2-n, 4, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{4}{3}n \text{AppellF1}[3, \right. \\
& \quad \left. 1+n, 1-n, 4, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right] + \frac{2}{3}(1+n) \text{AppellF1}[3, 2+n, -n, 4, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& \left(2 \text{AppellF1}[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2] + \right. \\
& \quad n \left(\text{AppellF1}[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2] + \text{AppellF1}[2, \right. \\
& \quad \left. 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 + \\
& \left(2(-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), \right. \\
& \quad \left. 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] \csc\left[\frac{1}{2}(e+fx)\right] \sec\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left((-1+n) \left(-2(-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\right. \right. \right. \\
& \quad \left. \left. \frac{1}{2}(e+fx)\right]^2] + \left(n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), \right. \\
& \quad \left. 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] - 2 \text{AppellF1}[2-n, -n, 2, 3-n, \frac{1}{2} \right. \\
& \quad \left. \left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] \right) \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(2(-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), \right. \\
& \quad \left. 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \\
& \left((-1+n) \left(-2(-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\right. \right. \right. \\
& \quad \left. \left. \frac{1}{2}(e+fx)\right]^2] + \left(n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), \right. \\
& \quad \left. 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] - 2 \text{AppellF1}[2-n, -n, 2, 3-n, \frac{1}{2} \right. \\
& \quad \left. \left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] \right) \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(2(-2+n) \cot\left[\frac{1}{2}(e+fx)\right]^2 \left(\frac{1}{2(2-n)}(1-n)n \text{AppellF1}[2-n, 1-n, 1, 3- \right. \right. \\
& \quad \left. \left. n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]-\frac{1}{2-\mathbf{n}}(1-\mathbf{n}) \operatorname{AppellF1}\left[2-\right. \\
& \quad \mathbf{n},-\mathbf{n},2,3-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2] \\
& \quad \left.\operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\left(-1+\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)\Bigg) / \\
& \quad \left((-1+\mathbf{n})\left(-2(-2+\mathbf{n}) \operatorname{AppellF1}[1-\mathbf{n},-\mathbf{n},1,2-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right), 1-\operatorname{Tan}\right.\right.\right. \\
& \quad \left.\left.\left.\left.\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\left(\mathbf{n} \operatorname{AppellF1}[2-\mathbf{n},1-\mathbf{n},1,3-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right.\right.\right. \\
& \quad \left.\left.\left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]-2 \operatorname{AppellF1}[2-\mathbf{n},-\mathbf{n},2,3-\mathbf{n}, \frac{1}{2}\right. \\
& \quad \left.\left.\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right)\left(-1+\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)\Bigg)- \\
& \quad \left(2(-2+\mathbf{n}) \operatorname{AppellF1}[1-\mathbf{n},-\mathbf{n},1,2-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right. \\
& \quad \left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Cot}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\left(-1+\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right) \\
& \quad \left.\left(\left(\mathbf{n} \operatorname{AppellF1}[2-\mathbf{n},1-\mathbf{n},1,3-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)-\right.\right. \\
& \quad \left.\left.2 \operatorname{AppellF1}[2-\mathbf{n},-\mathbf{n},2,3-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right.\right. \\
& \quad \left.\left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]-2(-2+\mathbf{n}) \\
& \quad \left(\frac{1}{2(2-\mathbf{n})}(1-\mathbf{n}) \mathbf{n} \operatorname{AppellF1}[2-\mathbf{n},1-\mathbf{n},1,3-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right. \\
& \quad \left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]-\frac{1}{2-\mathbf{n}}(1-\mathbf{n}) \\
& \quad \operatorname{AppellF1}[2-\mathbf{n},-\mathbf{n},2,3-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2] \\
& \quad \left.\operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+\left(-1+\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right) \\
& \quad \left(\mathbf{n}\left(-\frac{1}{3-\mathbf{n}}(2-\mathbf{n}) \operatorname{AppellF1}[3-\mathbf{n},1-\mathbf{n},2,4-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right.\right. \\
& \quad \left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]-\frac{1}{2(3-\mathbf{n})}\right. \\
& \quad \left.(1-\mathbf{n})(2-\mathbf{n}) \operatorname{AppellF1}[3-\mathbf{n},2-\mathbf{n},1,4-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right. \\
& \quad \left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\Big)- \\
& \quad 2\left(\frac{1}{2(3-\mathbf{n})}(2-\mathbf{n}) \mathbf{n} \operatorname{AppellF1}[3-\mathbf{n},1-\mathbf{n},2,4-\mathbf{n}, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right),\right. \\
& \quad \left.1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]-\frac{1}{3-\mathbf{n}}
\end{aligned}$$

$$\begin{aligned}
 & 2(2-n) \operatorname{AppellF1}\left[3-n, -n, 3, 4-n, \frac{1}{2} \left(1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right), \right. \\
 & \quad \left.\left.1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\right)\left.\right)\Bigg) \\
 & \left((-1+n)\left(-2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \left(1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right), \right.\right.\right. \right. \\
 & \quad \left.\left.\left.\left.1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \right.\right.\right. \\
 & \quad \left.\left.\left.\frac{1}{2} \left(1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - \right.\right.\right. \\
 & \quad \left.\left.\left.2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \left(1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right), \right.\right.\right. \\
 & \quad \left.\left.\left.1-\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right)\left(-1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)^2\right)\right)\Bigg)
 \end{aligned}$$

Problem 497: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \sec[e+f x])^n \sin[e+f x]^6 dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$- \left(\left(b \text{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+f x]^2\right] (b \sec[e+f x])^{-1+n} \sin[e+f x] \right) \right. \\
 \left. \left(f (1-n) \sqrt{\sin[e+f x]^2} \right) \right)$$

Result (type 6, 8327 leaves):

$$\begin{aligned}
 & \left(384 (b \sec[e+f x])^n \sin[e+f x]^6 \right. \\
 & \quad \left. \tan\left[\frac{1}{2}(e+f x)\right] \left(\frac{1}{1-\tan\left[\frac{1}{2}(e+f x)\right]^2}\right)^n \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)^{-7+n} \right. \\
 & \quad \left(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)^3 \right) \right. \\
 & \quad \left. \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 2 \right. \right. \\
 & \quad \left. \left. \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + n \operatorname{AppellF1}\left[\right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+f x)\right]^2 \right) - \right. \\
 & \quad \left. \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& 2 \left((-5+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \Big) \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2 \Big) + \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-6+n) \right. \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, n, 7-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. 1+n, 6-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) - \\
& \operatorname{AppellF1} \left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-7+n) \right. \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. 1+n, 7-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) + \\
& 192 \sec \left[\frac{1}{2} (e+f x) \right]^2 \left(\frac{1}{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2} \right)^n \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-7+n} \\
& \left(\left(\operatorname{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^3 \right) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-4+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2 - \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-5+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) + \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \\
& \left. \left. - \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right)\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + 2 \left((-6+n) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \left. \left. 1+n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right]\right) \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) - \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right]\right) / \right. \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + 2 \left((-7+n) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \left. \left. 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right]\right) \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) + \\
& 384 n \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \left(\frac{1}{1 - \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2}\right)^{1+n} \\
& \left(1 + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right)^{-7+n} \\
& \left(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right)^3\right)\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \\
& \left. 2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \right. \\
& \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right]\right) \right. \\
& \left. \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) - \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right)^2\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \\
& \left. 2 \left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \right. \\
& \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right]\right) \right. \\
& \left. \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) + \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \Bigg) \Bigg/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-6+n)\right.\right. \\
& \quad \left.\left.\operatorname{AppellF1}\left[\frac{3}{2}, n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2},\right.\right. \\
& \quad \left.\left.1+n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\Bigg) - \\
& \operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \Bigg/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-7+n)\right.\right. \\
& \quad \left.\left.\operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2},\right.\right. \\
& \quad \left.\left.1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\Bigg) + \\
& 384 \tan\left[\frac{1}{2}(e+fx)\right] \left(\frac{1}{1 - \tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^n \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^{-7+n} \\
& \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right.\right. \\
& \quad \left.\left.\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right)\right) \Bigg/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left.2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \quad \left.\left.n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right)\right. \\
& \quad \left.\tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \left(\left(-\frac{1}{3}(4-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2,\right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right.\right. \\
& \quad \left.\left.\frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right.\right. \\
& \quad \left.\left.\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^3\right)\right) \Bigg/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-4+n)\right.\right. \\
& \quad \left.\left.\operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2},\right.\right. \\
& \quad \left.\left.1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) - \\
& \left(6 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right)
\end{aligned}$$

$$\begin{aligned}
 & \left(-\frac{1}{3} (7-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right. \\
 & \quad \left. \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \right. \right. \\
 & \quad \left. \left. \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right) \Big/ \\
 & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \\
 & \quad 2 \left((-7+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \\
 & \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right) \right. \\
 & \quad \left. \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right) - \left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, \right. \right. \\
 & \quad \left. \left. -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \left(1 + \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right)^3 \right. \\
 & \quad \left. \left(2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right) \right. \\
 & \quad \left. \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] + 3 \left(-\frac{1}{3} (4-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] + \right. \\
 & \quad \left. \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right. \\
 & \quad \left. \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right) + 2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \\
 & \quad \left((-4+n) \left(-\frac{3}{5} (5-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right) \right. \right. \right. \\
 & \quad \left. \left. \left. \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right) + n \left(-\frac{3}{5} (4-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, \right. \right. \right. \\
 & \quad \left. \left. \left. 5-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\right. \right. \right. \\
 & \quad \left. \left. \left. \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] + \frac{3}{5} (1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. \left. -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right) \right) \right) \Big/ \\
 & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \\
 & \quad \left. 2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right. \\
& \quad \left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2+ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right. \\
& \quad \left.\left(1+\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)^2\left(2\left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right.\right. \\
& \quad \left.\left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+ \\
& \quad \left.3\left(-\frac{1}{3}(5-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right.\right. \\
& \quad \left.\left.\sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2},\right.\right.\right. \\
& \quad \left.\left.\left.\tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+\right. \\
& \quad \left.2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\left((-5+n)\left(-\frac{3}{5}(6-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 7-n, \frac{7}{2}, \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right.\right. \\
& \quad \left.\left.\left.\left.-\tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\right.\right.\right. \\
& \quad \left.\left.\left.\frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 6-n, \frac{7}{2}, \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right.\right.\right. \\
& \quad \left.\left.\left.\sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+n\left(-\frac{3}{5}(5-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n,\right.\right.\right. \\
& \quad \left.\left.\left.6-n, \frac{7}{2}, \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right.\right.\right. \\
& \quad \left.\left.\left.+\frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 5-n, \frac{7}{2}, \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right.\right. \\
& \quad \left.\left.\left.\left.-\tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\right)\right)\right)/ \\
& \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\right. \\
& \quad \left.2\left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right.\right. \\
& \quad \left.\left.n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)^2- \\
& \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right. \\
& \quad \left.\left(1+\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)\right. \\
& \quad \left.\left(2\left((-6+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right.\right.\right. \\
& \quad \left.\left.\left.+n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \sec \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\right)\right)
\end{aligned}$$

$$\begin{aligned}
& n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+3\left(-\frac{1}{3}(6-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 7-n, \frac{5}{2},\right.\right. \\
& \left.\left.\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\right. \\
& \left.\frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right. \\
& \left.\sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \\
& \left((-6+n)\left(-\frac{3}{5}(7-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 8-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\right. \\
& \left.\frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 7-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right. \\
& \left.\sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+n\left(-\frac{3}{5}(6-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n,\right.\right. \\
& \left.\left.7-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right. \\
& \left.\left.+\frac{3}{2}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\right)\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+ \right. \\
& \left.2\left((-6+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\right. \right. \\
& \left.\left.n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right. \right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2+ \right. \\
& \left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right. \\
& \left.2\left((-7+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\right. \right. \\
& \left.\left.n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right)\right. \\
& \left.\sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+3\left(-\frac{1}{3}(7-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2},\right.\right.\right. \\
& \left.\left.\left.\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\right. \right. \\
& \left.\left.\frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right)\right. \\
& \left.\sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2
\end{aligned}$$

$$\begin{aligned}
& \left((-7+n) \left(-\frac{3}{5} (8-n) \text{AppellF1}\left[\frac{5}{2}, n, 9-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] + \right. \\
& \quad \left. \left. \left. \frac{3}{5} n \text{AppellF1}\left[\frac{5}{2}, 1+n, 8-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right. \right. \\
& \quad \left. \left. \left. \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right) + n \left(-\frac{3}{5} (7-n) \text{AppellF1}\left[\frac{5}{2}, 1+n, \right. \right. \right. \\
& \quad \left. \left. \left. 8-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right. \right. \\
& \quad \left. \left. \left. + \frac{3}{5} (1+n) \text{AppellF1}\left[\frac{5}{2}, 2+n, 7-n, \frac{7}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \sec\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \right) \right) \right) \Bigg) / \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \\
& \quad \left. 2 \left((-7+n) \text{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] + \right. \right. \\
& \quad \left. \left. n \text{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right] \right) \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right)^2 \right) \Bigg)
\end{aligned}$$

Problem 498: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \sec[\mathbf{e} + \mathbf{f} x])^n \sin[\mathbf{e} + \mathbf{f} x]^4 dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\begin{aligned}
& - \left(\left(b \text{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[\mathbf{e} + \mathbf{f} x]^2\right] (b \sec[\mathbf{e} + \mathbf{f} x])^{-1+n} \sin[\mathbf{e} + \mathbf{f} x] \right) \right. \\
& \quad \left. \left(f (1-n) \sqrt{\sin[\mathbf{e} + \mathbf{f} x]^2} \right) \right)
\end{aligned}$$

Result (type 6, 6231 leaves):

$$\begin{aligned}
& \left(96 (b \sec[\mathbf{e} + \mathbf{f} x])^n \sin[\mathbf{e} + \mathbf{f} x]^4 \right. \\
& \quad \left. \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right] \left(\frac{1}{1 - \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2} \right)^n \left(1 + \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right)^{-5+n} \right. \\
& \quad \left. \left(\text{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2\right] \left(1 + \tan\left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x)\right]^2 \right)^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \right. \\
& \quad \left((-3+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\right. \right. \\
& \quad \left. \left. \frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 - \\
& \left(2 \operatorname{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \right. \\
& \quad \left((-4+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\right. \right. \\
& \quad \left. \left. \frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 + \\
& \operatorname{AppellF1} \left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \right. \\
& \quad \left((-5+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\right. \right. \\
& \quad \left. \left. \frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \Bigg) \Bigg) / \\
& \left(f \left(96 (-5+n) \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right]^2 \left(\frac{1}{1-\tan \left[\frac{1}{2} (e+f x) \right]^2} \right)^n \right. \right. \\
& \quad \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-6+n} \left(\left(\operatorname{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^2 \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-3+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \\
& \quad \left. \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) - \left(2 \operatorname{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-4+n) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. 1+n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& 1 + n, 4 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right) \tan\left[\frac{1}{2}(e + f x)\right]^2\Big) + \\
& \text{AppellF1}\left[\frac{1}{2}, n, 5 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] / \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 5 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + 2 \left((-5 + n) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, n, 6 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + n \text{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. 1 + n, 5 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e + f x)\right]^2\Big) + \\
& 48 \sec\left[\frac{1}{2}(e + f x)\right]^2 \left(\frac{1}{1 - \tan\left[\frac{1}{2}(e + f x)\right]^2}\right)^n \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right)^{-5+n} \\
& \left(\left(\text{AppellF1}\left[\frac{1}{2}, n, 3 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right)^2\right) / \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 3 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \\
& 2 \left((-3 + n) \text{AppellF1}\left[\frac{3}{2}, n, 4 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \\
& \left. n \text{AppellF1}\left[\frac{3}{2}, 1 + n, 3 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \\
& \tan\left[\frac{1}{2}(e + f x)\right]^2 - \left(2 \text{AppellF1}\left[\frac{1}{2}, n, 4 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \\
& \left. \left.-\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right)\right) / \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 4 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + 2 \left((-4 + n) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, n, 5 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + n \text{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. 1 + n, 4 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e + f x)\right]^2\right) + \\
& \text{AppellF1}\left[\frac{1}{2}, n, 5 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] / \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 5 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + 2 \left((-5 + n) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, n, 6 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + n \text{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. 1 + n, 5 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e + f x)\right]^2\right) + \\
& 96 n \sec\left[\frac{1}{2}(e + f x)\right]^2 \tan\left[\frac{1}{2}(e + f x)\right]^2 \left(\frac{1}{1 - \tan\left[\frac{1}{2}(e + f x)\right]^2}\right)^{1+n} \\
& \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right)^{-5+n}
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \right. \\
& \quad \left(3 \text{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-3+n) \text{AppellF1} \left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. n \text{AppellF1} \left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2 \left. \right) - \left(2 \text{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \quad \left(3 \text{AppellF1} \left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-4+n) \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \text{AppellF1} \left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. 1+n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) + \\
& \quad \left. \left(\text{AppellF1} \left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) / \right. \\
& \quad \left(3 \text{AppellF1} \left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-5+n) \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \text{AppellF1} \left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. 1+n, 5-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) + \\
& \quad 96 \tan \left[\frac{1}{2} (e+f x) \right] \left(\frac{1}{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2} \right)^n \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-5+n} \\
& \quad \left(\left(2 \text{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \right. \\
& \quad \left(3 \text{AppellF1} \left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-3+n) \text{AppellF1} \left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. n \text{AppellF1} \left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \\
& \quad \tan \left[\frac{1}{2} (e+f x) \right]^2 + \left(\left(-\frac{1}{3} (3-n) \text{AppellF1} \left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)^2\Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& 2\left((-3+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \\
& \tan\left[\frac{1}{2}(e+f x)\right]^2\Big) - \left(2 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right. \\
& \left. -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& 2\left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \\
& \tan\left[\frac{1}{2}(e+f x)\right]^2\Big) - \left(2\left(-\frac{1}{3}(4-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\Big) \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)\Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 2\left((-4+n) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+f x)\right]^2\Big) + \\
& \left(-\frac{1}{3}(5-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \\
& \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \right. \\
& \left. \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& 2\left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\Big)
\end{aligned}$$

$$\begin{aligned}
 & \left. \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) - \left(\text{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \\
 & \left. \left. - \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right)^2 \right. \\
 & \left. \left(2 \left((-3+n) \text{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \right. \right. \\
 & \left. \left. \left. n \text{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right]\right) \right. \\
 & \left. \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right] + 3 \left(-\frac{1}{3}(3-n) \text{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \right. \right. \right. \\
 & \left. \left. \left. \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \right. \\
 & \left. \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right] + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \right. \\
 & \left. \frac{1}{3}n \text{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \right. \\
 & \left. \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]\right) + 2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \\
 & \left. \left((-3+n) \left(-\frac{3}{5}(4-n) \text{AppellF1}\left[\frac{5}{2}, n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right] + \right. \right. \right. \\
 & \left. \left. \left. \left. \frac{3}{5}n \text{AppellF1}\left[\frac{5}{2}, 1+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]\right) + n \left(-\frac{3}{5}(3-n) \text{AppellF1}\left[\frac{5}{2}, 1+n, \right. \right. \right. \right. \\
 & \left. \left. \left. \left. 4-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right] \right. \right. \right. \right. \\
 & \left. \left. \left. \left. + \frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right] + \frac{3}{5}(1+n) \text{AppellF1}\left[\frac{5}{2}, 2+n, 3-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \right. \right. \\
 & \left. \left. \left. \left. -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]\right)\right)\right) \right) \Bigg) \Bigg) \\
 & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \\
 & \left. 2 \left((-3+n) \text{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + \right. \right. \\
 & \left. \left. n \text{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right)^2 + \right. \\
 & \left. \left(2 \text{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \right. \\
 & \left. \left(1 + \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right) \left(2 \left((-4+n) \text{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] + n \text{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. -\tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e} + \mathbf{f}x)\right] + \right. \right. \right. \right. \right)
 \end{aligned}$$

$$\begin{aligned}
& 3 \left(-\frac{1}{3} (4-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right. \\
& \quad \left. \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] \right) + \\
& 2 \tan\left[\frac{1}{2} (e+f x)\right]^2 \left((-4+n) \left(-\frac{3}{5} (5-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] \right) + \right. \\
& \quad \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right. \\
& \quad \left. \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] \right) + n \left(-\frac{3}{5} (4-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, \right. \right. \\
& \quad \left. \left. 5-n, \frac{7}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] \right. \\
& \quad \left. + \frac{3}{5} (1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] \right) \right) \Bigg) \Bigg) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] + \right. \\
& \quad 2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] + \right. \\
& \quad \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right) \tan\left[\frac{1}{2} (e+f x)\right]^2 - \\
& \quad \left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right. \\
& \quad \left(2 \left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] + \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right) \right. \\
& \quad \left. \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] + 3 \left(-\frac{1}{3} (5-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] + \right. \\
& \quad \left. \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right. \\
& \quad \left. \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] \right) + 2 \tan\left[\frac{1}{2} (e+f x)\right]^2 \\
& \quad \left((-5+n) \left(-\frac{3}{5} (6-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 7-n, \frac{7}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] + \right. \right. \\
& \quad \left. \left. \left. \frac{1}{2} (e+f x)\right]^2 \right) \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] + \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\Big) + n\left(-\frac{3}{5}(5-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n,\right.\right. \\
& \left.\left.6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right. \\
& \left.\left.+\frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right. \\
& \left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\Big)\Big)\Big) \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] +\right. \\
& \left.2\left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] +\right.\right. \\
& \left.\left.n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right. \\
& \left.\left.-\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)^2\right)\Big)
\end{aligned}$$

Problem 499: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (\mathbf{b} \sec[\mathbf{e}+\mathbf{f} x])^n \sin[\mathbf{e}+\mathbf{f} x]^2 dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\begin{aligned}
& -\left(\left(\mathbf{b} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[\mathbf{e}+\mathbf{f} x]^2\right] (\mathbf{b} \sec[\mathbf{e}+\mathbf{f} x])^{-1+n} \sin[\mathbf{e}+\mathbf{f} x]\right) / \right. \\
& \left.\left(\mathbf{f} (1-n) \sqrt{\sin[\mathbf{e}+\mathbf{f} x]^2}\right)\right)
\end{aligned}$$

Result (type 6, 4143 leaves):

$$\begin{aligned}
& \left(24\left(\sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)^{-3+n} (\mathbf{b} \sec[\mathbf{e}+\mathbf{f} x])^n\right. \\
& \left.\left(\cos\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \sec[\mathbf{e}+\mathbf{f} x]\right)^n \sin[\mathbf{e}+\mathbf{f} x]^2 \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right. \\
& \left.\left(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)\right)\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] + 2\right. \\
& \left.\left(\left(-2+n\right) \operatorname{AppellF1}\left[\frac{3}{2}, n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] + n \operatorname{AppellF1}\left[\right.\right.\right. \\
& \left.\left.\left.\frac{3}{2}, 1+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right) - \\
& \left.\left.\left.\operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} (\text{e} + \text{f} x) \Big] + \frac{3}{5} (1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 3-n, \frac{7}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2,\right. \\
& \quad \left. -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2 \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]\Big)\Big)\Big)\Big) \Big/ \\
& \Big(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] + \\
& \quad 2 \left((-3+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] + \right. \\
& \quad \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2,\right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right)^2 \Big) + \\
& 24 n \left(\sec\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2 \right)^{-3+n} \left(\cos\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2 \sec[\text{e} + \text{f} x] \right)^{-1+n} \\
& \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right] \\
& \Big(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] \sec\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2 \right) \Big/ \\
& \Big(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] + \\
& \quad 2 \left((-2+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] + \right. \\
& \quad \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2,\right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right) - \\
& \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] \Big/ \\
& \Big(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] + \\
& \quad 2 \left((-3+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2, -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] + \right. \\
& \quad \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2,\right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right] \tan\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2\right) \Big) \\
& \Big(-\cos\left[\frac{1}{2}(\text{e} + \text{f} x)\right] \sec[\text{e} + \text{f} x] \sin\left[\frac{1}{2}(\text{e} + \text{f} x)\right] + \cos\left[\frac{1}{2}(\text{e} + \text{f} x)\right]^2 \\
& \quad \sec[\text{e} + \text{f} x] \tan[\text{e} + \text{f} x] \Big) \Big) \Big)
\end{aligned}$$

Problem 501: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc[e + fx]^2 (b \sec[e + fx])^n dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$-\frac{1}{f(1-n)} b \csc[e + fx] \\ \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e + fx]^2\right] (b \sec[e + fx])^{-1+n} \sqrt{\sin[e + fx]^2}$$

Result (type 6, 3228 leaves):

$$\begin{aligned} & \left(\cot\left(\frac{1}{2}(e + fx)\right) \csc[e + fx]^2 \sec[e + fx]^n (b \sec[e + fx])^n \right. \\ & \left(-\left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right. \right. \\ & \quad \left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \right. \\ & \quad 2n \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \text{AppellF1}\left[\frac{1}{2}, \right. \right. \\ & \quad \left. \left. 1+n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) + \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) \right. \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \right. \\ & \quad 2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, \right. \right. \\ & \quad \left. \left. 1+n, -n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) \right) \right. \\ & \quad \left(2f \left(-\frac{1}{4} \csc\left[\frac{1}{2}(e + fx)\right]^2 \sec[e + fx]^n \left(-\left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right. \right. \right. \\ & \quad \left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + 2n \right. \\ & \quad \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \text{AppellF1}\left[\frac{1}{2}, \right. \right. \\ & \quad \left. \left. 1+n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) \right) + \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) \right. \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \right. \\ & \quad 2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, \right. \right. \\ & \quad \left. \left. 1+n, -n, \frac{5}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) + \end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{1}{3} \mathbf{n} \operatorname{AppellF1}\left[\frac{3}{2}, 1+\mathbf{n},-\mathbf{n}, \frac{5}{2},\right. \\
& \left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+ \\
& 2 \mathbf{n} \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\left(-\frac{3}{5}(1-\mathbf{n}) \operatorname{AppellF1}\left[\frac{5}{2}, \mathbf{n}, 2-\mathbf{n}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{6}{5} \mathbf{n} \operatorname{AppellF1}\left[\frac{5}{2},\right.\right. \\
& \left.\left.1+\mathbf{n}, 1-\mathbf{n}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{3}{5}(1+\mathbf{n}) \operatorname{AppellF1}\left[\frac{5}{2}, 2+\mathbf{n},-\mathbf{n}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\right)\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, \mathbf{n},-\mathbf{n}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+2 \mathbf{n}\right. \\
& \left.\left(\operatorname{AppellF1}\left[\frac{3}{2}, \mathbf{n}, 1-\mathbf{n}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\operatorname{AppellF1}\left[\frac{3}{2},\right.\right.\right. \\
& \left.\left.\left.1+\mathbf{n},-\mathbf{n}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)\right)
\end{aligned}$$

Problem 502: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc [\mathbf{e}+\mathbf{f} x]^4(b \sec [\mathbf{e}+\mathbf{f} x])^{\mathbf{n}} dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\begin{aligned}
& -\frac{1}{\mathbf{f}(1-\mathbf{n})} b \csc [\mathbf{e}+\mathbf{f} x] \\
& \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1-\mathbf{n}}{2}, \frac{3-\mathbf{n}}{2}, \cos [\mathbf{e}+\mathbf{f} x]^2\right] (b \sec [\mathbf{e}+\mathbf{f} x])^{-1+\mathbf{n}} \sqrt{\sin [\mathbf{e}+\mathbf{f} x]^2}
\end{aligned}$$

Result (type 6, 6799 leaves):

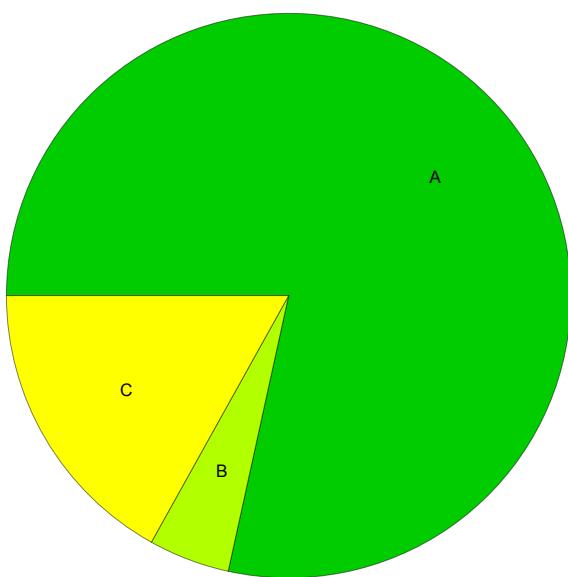
$$\begin{aligned}
& \left(\cot \left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^3 \csc [\mathbf{e}+\mathbf{f} x]^4(b \sec [\mathbf{e}+\mathbf{f} x])^{\mathbf{n}}\left(\frac{1+\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2}\right)^{\mathbf{n}}\right. \\
& \left.\left(-\left(\operatorname{AppellF1}\left[-\frac{3}{2}, \mathbf{n},-\mathbf{n},-\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) / \right.\right. \\
& \left.\left.\left(\operatorname{AppellF1}\left[-\frac{3}{2}, \mathbf{n},-\mathbf{n},-\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]-\right.\right.\right. \\
& 2 \mathbf{n}\left(\operatorname{AppellF1}\left[-\frac{1}{2}, \mathbf{n}, 1-\mathbf{n}, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\operatorname{AppellF1}\left[-\frac{1}{2},\right.\right. \\
& \left.\left.1+\mathbf{n},-\mathbf{n}, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right)-\right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\Big)\Big)\Big)\Big) \\
& \left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& 2n \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \left(27 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \tan\left[\frac{1}{2}(e+fx)\right]^4 \left(2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& 3 \left(\frac{1}{3}n \text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3}n \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \right. \\
& \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \Big) + \\
& 2n \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{3}{5}(1-n) \text{AppellF1}\left[\frac{5}{2}, n, 2-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{6}{5}n \text{AppellF1}\left[\frac{5}{2}, \right. \right. \\
& \left. \left. 1+n, 1-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \tan\left[\frac{1}{2}(e+fx)\right] + \frac{3}{5}(1+n) \text{AppellF1}\left[\frac{5}{2}, 2+n, -n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big) \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& 2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \left(5 \text{AppellF1}\left[\frac{3}{2}, n, -n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^6 \right. \\
& \left. \left(2n \left(\text{AppellF1}\left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{5}{2}, 1+n, -n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right)
\end{aligned}$$

$$\begin{aligned}
& 5 \left(\frac{3}{5} n \operatorname{AppellF1} \left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] + \frac{3}{5} n \operatorname{AppellF1} \left[\frac{5}{2}, 1+n, -n, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \right) + \\
& 2 n \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \left(-\frac{5}{7} (1-n) \operatorname{AppellF1} \left[\frac{7}{2}, n, 2-n, \frac{9}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] + \frac{10}{7} n \operatorname{AppellF1} \left[\frac{7}{2}, \right. \right. \\
& \quad \left. \left. 1+n, 1-n, \frac{9}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right. \\
& \quad \left. \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] + \frac{5}{7} (1+n) \operatorname{AppellF1} \left[\frac{7}{2}, 2+n, -n, \frac{9}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \sec \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \right) \right) \Bigg) \Bigg) \Bigg) \\
& \left(5 \operatorname{AppellF1} \left[\frac{3}{2}, n, -n, \frac{5}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] + 2 n \right. \\
& \quad \left. \left(\operatorname{AppellF1} \left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] + \operatorname{AppellF1} \left[\frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 1+n, -n, \frac{7}{2}, \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right)^2 \right) \right)
\end{aligned}$$

Summary of Integration Test Results

538 integration problems



A - 422 optimal antiderivatives

B - 25 more than twice size of optimal antiderivatives

C - 91 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts